# PROGRAMME: B.Sc. PHYSICS HONOURS

## SEMESTER - V

## Course: CLASSICAL DYNAMICS (DSE-2)

## Lecture Note: 1

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# Classical Mechanics:

## Constraints and Constrained Motion:-

The motion of a particle in real life is restricted or limited or constrained. To move in someway such that it's co-ordinate or the velocity component must maintain a given relation at every instant of time. This limitation for geometrical restriction on the particle or the the system of particle are generally known as constraint. In a word the limitations on the motion of a system are called constraints and the motion of the system is said to be constrained motion. Physically a constrained motion is realised by the forces which arise when the particle in motion is in contact with the constraining surface or curves. These forces are called Constraint forces. The basic properties of these forces are

i) They are elastic in nature and appear at the surface of contact.

ii) They are so strong that they barely allow the body under consideration to deviate even slightly from a prescribed path. This prescribed path or surface is called a constraint surface. The scalar equation that describe the surface of constraint are called Constraint equations.

## Classification of Constraints :-

The constraints are

i) a) Scleronomic :- Constraint relation do not explicitly depend on time.

b) Rheonomic :- constraint reation depend explicitly on time.

ii) a) Holonomic :- The constraint relation are or can be made independent of velocities $(\dot{x}, \dot{y}, \dot{z}, \dot{\theta} \ldots)$

b) Nonholonomic :- Constraint relations are not holonomic.

iii) a) Conservative :- If the total mechanical energy of the system is conserved while performing the constraint motion and constraint forces do not do any work, then constraint is said to be conservative.

b) Dissipative :- If the constraint force do work and the total energy

is

iv)

Fir

i)

[redacted]

is not conserved, it is dissipative Constraint.

iv) a) Bilateral :- At any point on the Constraint Surface both the forward and the backward motions are possible. Constraint relation are not in the form of inequalities but are in the form of equations.

$$f(\bar{r}_1, \bar{r}_2, \bar{r}_3, \cdots t) = 0$$

b) Unilateral :→ The constraint equation is in the form of inequalities.

Find the Constraints in following motions :-

i) Rigid Body Motion :-
In case of rigid body motion distance between any two particles of the body remains fixed and do not Change with time. If $\vec{r}_i$ and $\vec{r}_j$ are the position vector of the ith and jth particles, then the distance between them can be expressed by

$$|\vec{r}_i - \vec{r}_j| = \text{constant.} \quad \text{—} \; (1)$$

a) As the constraint relation of (1) do not depend on time, so, it is Scleronomic constraint.

b) The constraint relation is independent of velocity. So it is Holonomic Constraint.

c) The constraint relation is in the

[redacted]

is considered parallel to the line joining ith and jth particles of the form

$$\vec{F}_{ij} = c_{ij}(\vec{r}_i - \vec{r}_j), \text{ where } c_{ij} = \text{constant.}$$

Here the work done by constraint forces in a rigid body is zero and consequently the constraint is conservative in nature.

ii) **Motion of deformable bodies :-**

Deformable bodies means whose shape can change with time. In this case the constrained ~~eq~~ equation is given by

$$|\vec{r}_i - \vec{r}_j| = f(t)$$

So this constraint relation can not give the ~~to~~ total work done $(W) = 0$.
So the constraint is ~~cons~~ dissipative one. The constraint is also ~~scleronomic~~ rheonomic, holonomic, bilateral

iii) **Motion of Simple Pendulum :-**

The position of the bob at any point must satisfy

$$|\vec{r}|^2 = x^2 + y^2 + z^2 = l^2 \text{ or } \boxed{\vec{r} \cdot d\vec{r} = 0} \quad (1)$$

where '$l$' is the constant length of the string connecting the bob to the fulcrum (which is taken to be origin)

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The tension in string $\vec{T}$ is parallel to $-\vec{r}_i$ giving the work done by the constraint force

$$\Delta W = \vec{T} \cdot \Delta\vec{r} = 0 \quad [\because \bar{r} \cdot d\bar{r} = 0]$$

So The constraints are <u>scleronomic</u>, <u>holonomic</u>, <u>bilateral</u> and <u>conservative</u>.

iv) <u>Pendulum with variable length</u>:-

Suppose the length of the string is changing according to a given function $l(t)$. Then the constraint relation is given by

$$|\bar{r}(t)|^2 = l^2(t) \quad \text{—— (1)}$$

or $\bar{r} \cdot d\bar{r} \neq 0$

In this case the work done

$$\Delta W = \bar{T} \cdot d\bar{r} \neq 0$$

So the constraint force do work.

So from (1) we found the constraint is <u>dissipative</u>, <u>rheonomic</u>, <u>bilateral</u>, <u>holonomic</u>.

v) <u>Motion of gas particle in a spherical container</u>:→

The constraint relation for the gas particle inside the container are $|\vec{r}_i| \leq R$ —— (1)

where $\vec{r}_i$ is measured from the centre of the cylinder.

'R' is the radious of the ~~cylinder~~ container.

In this case the constraint are Selesonomic, holonomic, conservative and unilateral.

VI) Rolling Without Sliding :-

Suppose a spherical ball or a cylinder is rolling on a plane without sliding. We assume that the surface in contact is perfectly rough. Thus frictional forces are not negligible. Since the point of contact is not sliding, the frictional force do not any work, and hence the total mechanical energy of the rolling body is conserved. Thus the constraint is conservative. To obtain the constraint equation we note that rolling without sliding means that the relative velocity of the point of contact with respect to the plane is zero. Then the velocity $\vec{V}$ of any point P in the rolling body, as seen ~~as~~ from a fixed frame of reference is given by

$$\vec{V} = \vec{V}_{cm} + \vec{\omega} \times \vec{r}$$

$\vec{V}_{cm}$ is the velocity of the centre of mass.

~~Since there is no sliding of this point~~

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~~We must have the instantaneous velocity v at the contact is zero.~~

$$\cancel{\vec{v} = \vec{v}_{cm} - r(\vec{\omega}\times\hat{n}) = 0}$$

~~Here~~ $\vec{r}$ is measured from the cm of the cylinder ~~of~~ or sphere.

The velocity of point of contact is obtained putting $\vec{r} = -r\hat{n}$ where $\hat{n}$ is the unit vector along the outward normal to the plane and $r$ is the radius ~~vector~~ of the sphere ~~of~~ or cylinder.

Since there is no sliding of this point we must have the instantaneous velocity $\vec{v}$ at the contact

$$\vec{v} = \vec{v}_{cm} - r(\vec{\omega}\times\hat{n}) = 0$$

or $\frac{dR}{dt} = 0$, $R = \text{const.}$

So the position of the sphere is const.

So the constraints are ~~hollo~~ holonomic, ~~stat~~ scleronomic, conservative, bilateral.

## Degrees of Freedom :→

The minimum number of independent ways in which a mechanical system can move without violating any constraints which may be imposed on the system is called the degrees of freedom. In other words,

It ~~that~~ is the number of independent parameters or variables necessary to describe the configuration of the dynamical system. If N is the number of particle in the system and K is the number of condition of holonomic constraints imposed on it, then degree of freedom of the system

$$\boxed{f = 3N - K} = \rightarrow 3D$$

$$= 2N - K \rightarrow 2D$$

* Find the degree of freedom of following :→

i) A particle moving on the circumference of a circle :→

The constraint is $x^2+y^2=a^2$ or $r = a$ (constant). Hence in cartesian co-ordinates one variable x or y and in polar co-ordinates one variable $\theta$ could suffice to describe the motion on the circle. Hence the degree of freedom is 1.

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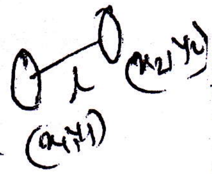
ii) Five particles moving freely in a plane :→

Each free particle needs two co-ordinates to specify its position in a plane. Hence 5 free particles will need ~~4~~ 10 co-ordinates and the ~~4~~ number of degree of freedom of the system is 10.

iii) Two particle connected by a rigid rod moving freely in a plane:→

Degree of freedom = $2\times2-1=3$
because two particles in a plane will need $[(x_1,y_1)$ and $(x_2,y_2)]$ 4 co-ordinates and there is one constraint equation




$$(x_2-x_1)^2+(y_2-y_1)^2=l^2$$

iv) A rigid body moving freely in three-dimensional space :-

A rigid body is a system of particles in which the distance between any two particles remain fixed throughout the motion. Let us consider three non-linear particles $P_1, P_2, P_3$ of a rigid body. The distance between ~~each~~ each three ~~part~~ particle remain fixed. So there are ~~to~~ 3 constraint

So no of D.O.F = $3\times3-3=6$.

v) A rigid body moving in space with one point fixed :–

One point of the rigid body is fixed, say the point at the origin. Hence for this particle $x_1 = 0$, $y_1 = 0$, $z_1 = 0$.
The constraint equation for other particles are

$$x_2^2 + y_2^2 + z_2^2 = r_2^2, \quad x_3^2 + y_3^2 + z_3^2 = r_3^2$$

and $$(x_2 - x_3)^2 + (y_2 - y_3)^2 + (z_2 - z_3)^2 = r_{23}^2 \text{ (const.)}$$

Hence the degree of freedom for the system are $3\times3 - 6 = 3$

vi) A rigid body rotating about a fixed axis in space :→

A rigid body, rotating about a fixed axis, has one degree of freedom because ~~the motion is described by~~ relative to origin, say on the fixed axis, taken as z axis $z = $ const. and $x^2 + y^2 = r_0^2$ for a particle where $r_0$ is the radius of the circle about the fixed axis.

vii) The bob of the simple pendulum oscillating in a plane :→

The bob of simple pendulum oscillating in a plane has one degree of freedom because the motion is described by one $\theta$ coordinate with the constraint $|\vec{r}| = l$

or in Cartesian coordinate by $x$ or $y$.
with the constraint $x^2+y^2=l^2$ (const)
~~and [illegible]~~ $=0$

viii) The bob of a conical pendulum :-

The bob of a conical pendulum has 2 degree of freedom as the constraint is $x^2+y^2+z^2=l^2$ relative to the centre of suspension.

(ix) Dumbell moving in space :-

In a dumbell two heavy particles are connected by a rigid rod. The system has

$3\times2-1=5$ degree of freedom.

because there is one constraint that the distance between two particle remain fixed.

$$(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2=\text{Constant.}$$

(x) The fixed fulcrum of a simple pendulum :- This is a point fixed on a ceiling. It requires three co-ordinates say $(x_0, y_0, z_0)$ to represent it position. But all these co-ordinates are fixed, not variable. So constraint relation

$$\left.\begin{aligned} x &= x_0 \\ y &= y_0 \\ z &= z_0 \end{aligned}\right\}$$

So No. of D.o.f $= 3\times1-3=0$