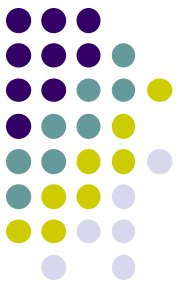
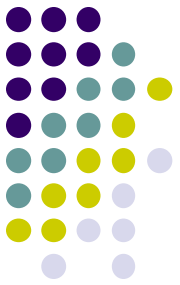


Syllabus



- **Electrodynamics:** Maxwell's equations: differential and integral forms, significance of Maxwell's equations, displacement current and correction in Ampere's law, electromagnetic wave propagation, transverse nature of EM waves, wave propagation in bounded system, applications.
- **Quantum Physics:** Dual nature of matter, de-Broglie Hypothesis, Heisenberg uncertainty principle and its applications, postulates of quantum mechanics, wave function & its physical significance, probability density, Schrodinger's wave equation, Eigen values & Eigen functions, Applications of Schrodinger equation.

OUTLINE



Atom and its size

Waves and Particles

Waves as particles and Particles as Waves
– Quantum View

Milestones of Quantum physics

Wave nature of Matter : de Broglie

Interference of waves

Heisenberg's uncertainty principle

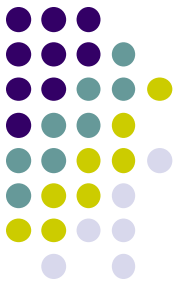
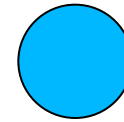
Quantum versus Classical world view

How big are atoms ?

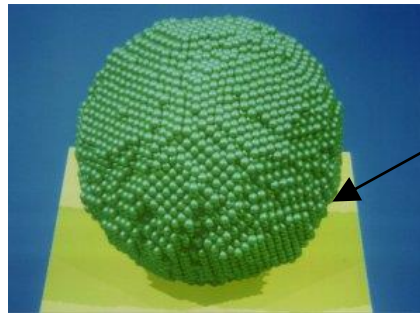


Democritus : Atoms as building blocks.

Size?
Shape ?
Substance?



17000 Copper atoms



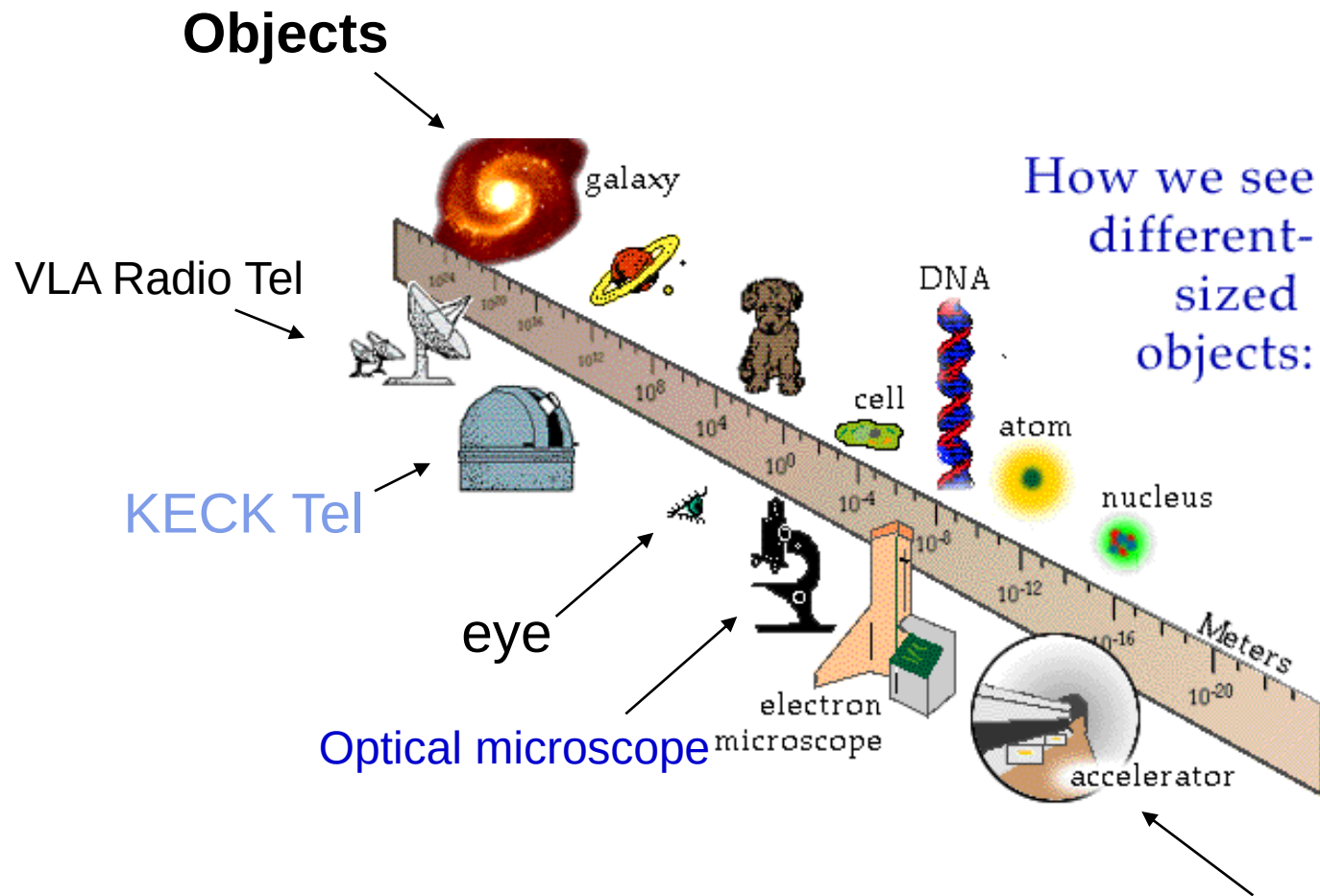
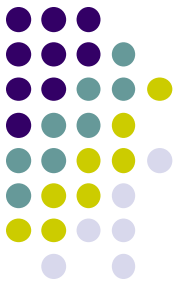
Diameter 10^{-7} cm

1 nm = 10^{-9} meters

Atomic size determined not till the 19th century

Atoms are very small ; about 0.5 nanometers.

Nanotechnology deals with atomic manipulations.

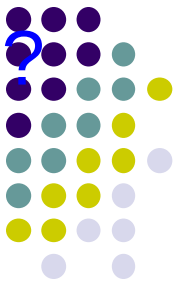


Techniques of observation

<http://www.vendian.org/howbig/>

Helps you visualize sizes

Waves and Particles : What do we mean by them?



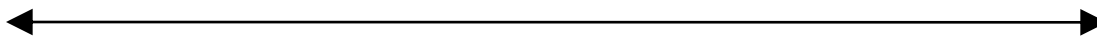
Material Objects:

Ball, Car, person, or point like objects called **particles**.
They can be located at a space point at a given time.
They can be at rest, moving or accelerating.

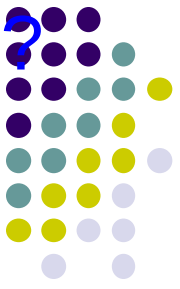
Falling Ball



Ground level



Waves and Particles: What do we mean by them?

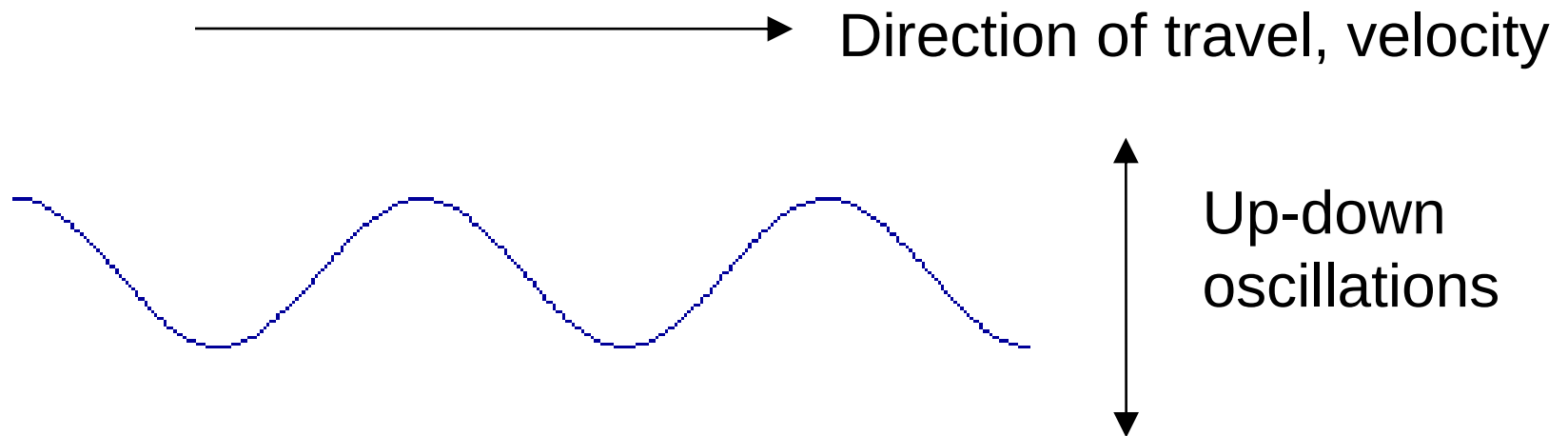


Common types of waves:

Ripples, surf, ocean waves, sound waves, radio waves.

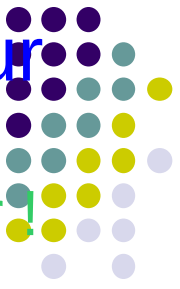
Need to see crests and troughs to define them.

Waves are oscillations in space and time.

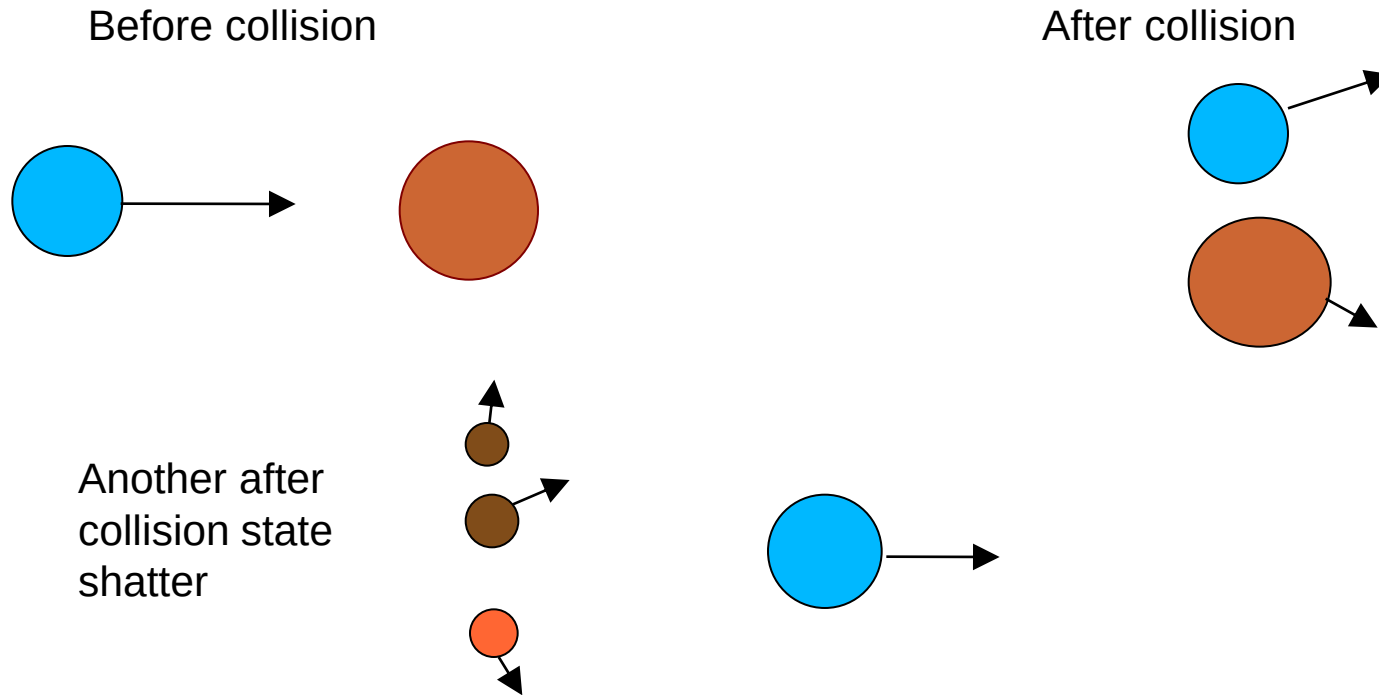


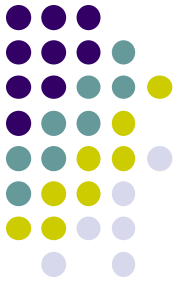
Wavelength ,frequency, velocity and oscillation size defines waves

Particles and Waves: Basic difference in behaviour



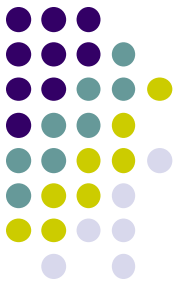
When particles collide they cannot pass through each other!
They can bounce or they can shatter





Collision of truck with ladder on top with a Car at rest ! Note the ladder continue its Motion forward Also the small car front End gets smashed.



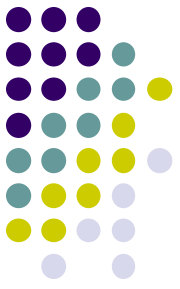


Head on collision of a car and truck
Collision is inelastic – the small car is dragged along
By the truck.....

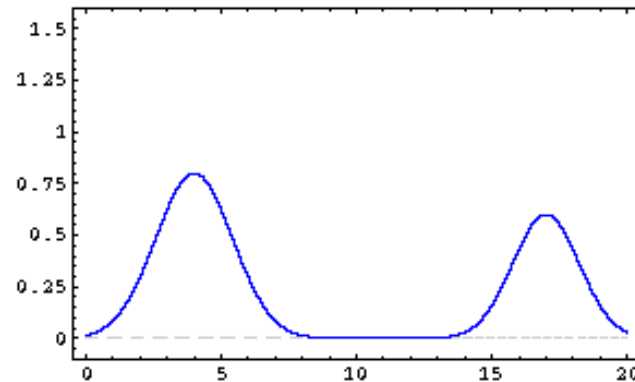
Car		Truck	
mass (kg)	1000	mass (kg)	3000
vel. (m/s)	20.0	vel. (m/s)	-20.0
mom. (kg m/s)	20 000	mom. (kg m/s)	-60 000



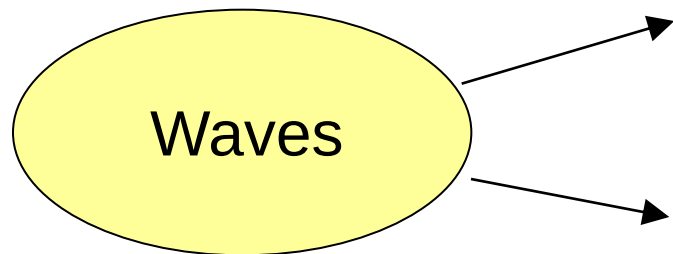
Waves and Particles Basic difference:



- Waves can pass through each other !
- As they pass through each other they can enhance or cancel each other
- Later they regain their original form !



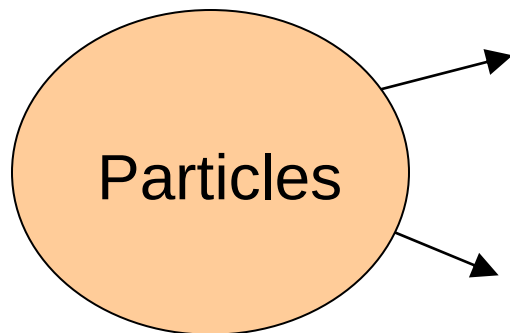
Waves and Particles:



Spread in space and time

Can be superimposed – show interference effects

Pass through each other



Localized in space and time

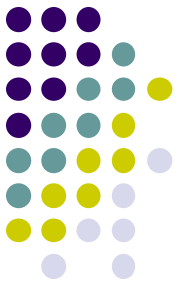
Cannot pass through each other - they bounce or shatter.

Wavelength

Frequency



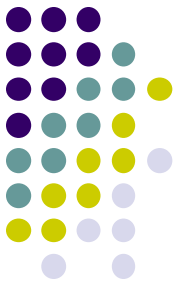
OUR QUANTUM WORLD



In the 20th century, study of atomic systems required a fundamental revision of these classical ideas about physical objects.

1. Light waves exhibited particle like properties –
phenomena called photo-electric effect in which light impinging on certain metals cause instantaneous emission of electrons in a billiard ball like impact.
– the basis of automatic door openers in grocery stores
2. Electrons (particles) exhibit wave like properties –
they can pass through each other !
Phenomenon of electron interference
– basis of electron microscopes

OUR QUANTUM WORLD

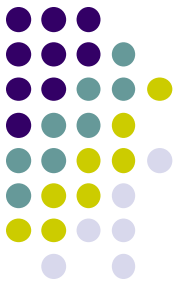


■

This quantum picture of the world is at odds with our common sense view of physical objects. We cannot uniquely define what is a particle and what is a wave !!

Neils Bohr and Werner Heisenberg were the architects of this quantum world view, along with Planck, Einstein, de Broglie, Schrodinger, Pauli and Dirac.

True understanding of nature required that physical objects, whatever they are, are neither exclusively particles or waves

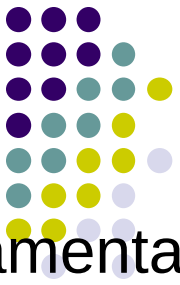


No experiment can ever measure both aspects at the same time, so we never see a mixture of particle and wave.

When one observes a physical phenomenon Involving a physical object, the behaviour You will observe – whether particle like or Wave like – depends on your method of Observation.

The object is described by mathematical Functions which are measures of probability .

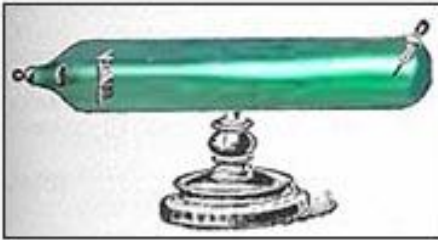
MILESTONES OF QUANTUM PHYSICS:



THOMSON

J.J.Thomson Established electron as a fundamental particle of nature. He measured its charge to mass ratio using a Crooke's tube.

Electric current = flow of electrons



Crooke's tube:
Evacuated tube
Visualization of
electron beam.

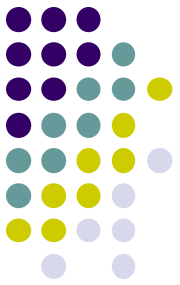


Animation of electrons moving
and being deflected by an electric
or magnetic field.



Marie Curie and Radioactivity - 1898

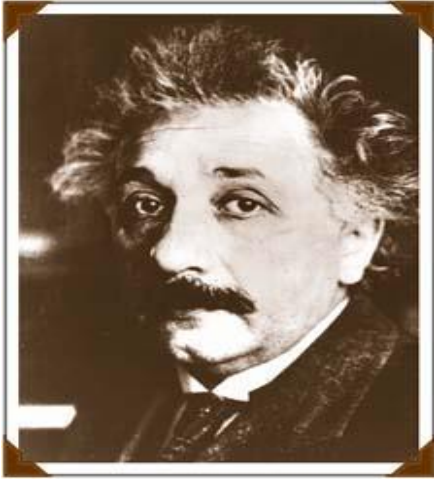
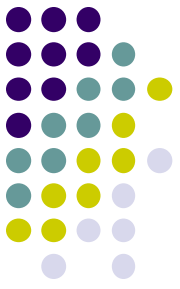
Discovered that certain elements 'spontaneously emit radiations and change into different elements.



Only woman scientist to receive two Nobel Prizes:
One in chemistry and the other in physics.

Marie Curie received the Nobel Prize in Physics with her husband Pierre Curie and Henri Becquerel in 1903 , for their work on radioactivity. In 1911, she won her second Nobel Prize, this time in chemistry for her work on polonium and radium.

The Quantum of Light or the Photon



Particle nature of light was proposed by Einstein in 1905 to explain the photo-electric effect. Photo-electric effect – automatic door openers in grocery stores. **Particles of light are called light quanta or photons.**

Energy of a Photon = $h \times \text{frequency of light}$
 h is a fundamental constant of nature and it is very small in size.

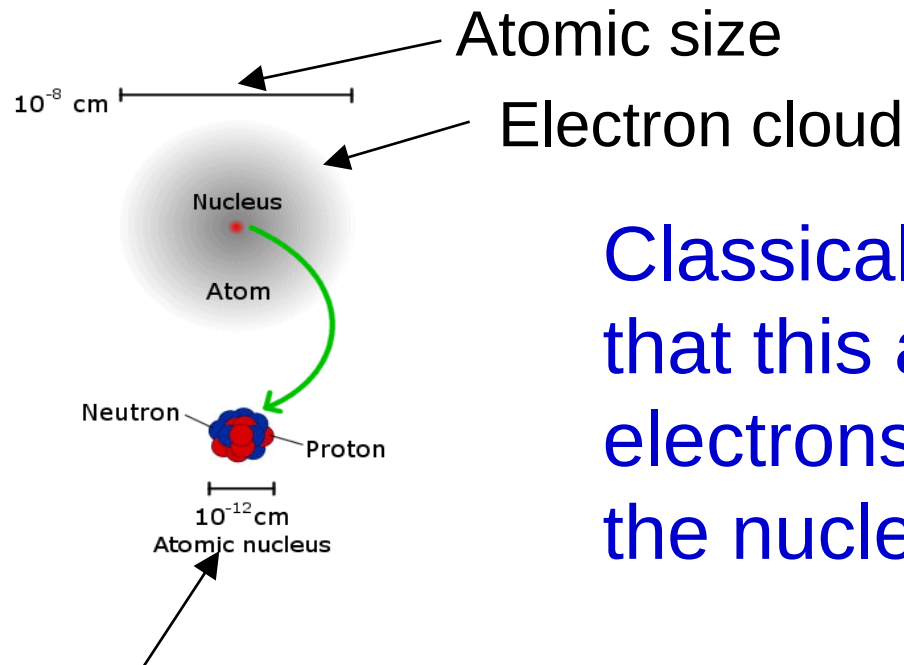
Packet of energy in photon is so small that we are not aware of the rain of photons of light impinging on our eyes – just as you cannot feel the impact of individual air molecules, you only feel a breeze.

Rutherford and his Nuclear Atom: 1898 -1911



RUTHERFORD

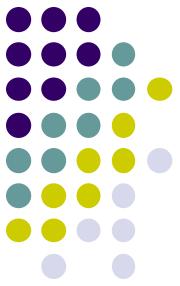
Ernest Rutherford used alpha rays to discover the nucleus of the atom. The nucleus was positively charged and contained almost all of the mass of the atom. Most of the atom was empty space.



Classical physics required that this atom is unstable electrons would fall into the nucleus in 10^{-7} sec!



Planck and quantization of atomic “vibrations”



Before Einstein, Planck postulated from study of radiation from hot bodies that the radiating atoms can only radiate energy in discrete amounts – or that atoms exist only in discrete states, called Quantum states.

This was the birth of quantum physics in 1900

THE BOHR ATOM:

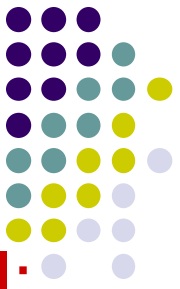


BOHR

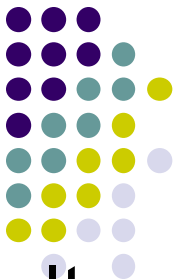
Bohr proposed a revolutionary model:
An atom with discrete (Quantum) states
– an ad-hoc model

Bohr model explained how atoms emit light quanta and their stability. He combined the postulates of Planck and Einstein to build characteristic energy states that atoms should possess.

Model gave excellent agreement with experiment on atomic spectra.(1913)



Bohr atom



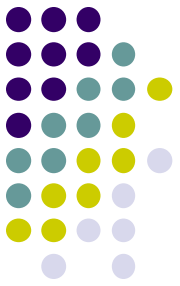
Bohr's atom model achieved three important results:

1. Atoms are stable
2. Different atoms of the same element are identical
3. Atoms regenerate if they are taken apart and then allowed to reform.

THE BOHR ATOM:

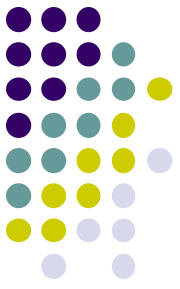


BOHR



Understanding the origin of Bohr's model required an essential bold step – enter Louis de Broglie.

Wave nature of material bodies:

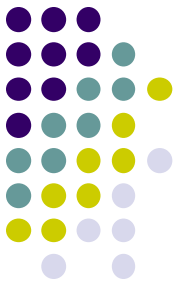


If light, which classically is a wave, can have particle nature As shown by Planck and Einstein,

Can material particles exhibit wave nature ?

Prince Louis de Broglie while doing his Ph.D. research said particles should have wave like properties.

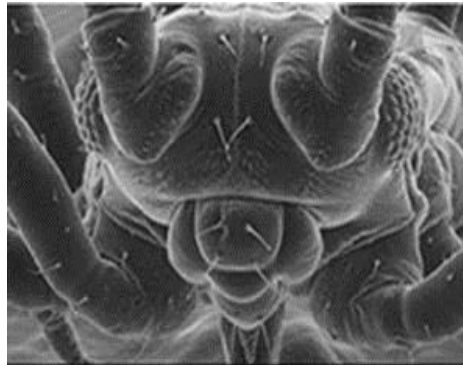
Wave Nature of Matter



Louis de Broglie in 1923 proposed that matter particles should exhibit wave properties just as light waves exhibited particle properties. These waves have very small wavelengths in most situations so that their presence was difficult to observe

These waves were observed a few years later by Davisson and G.P. Thomson with high energy electrons. These electrons show the same pattern when scattered from crystals as X-rays of similar wave lengths.

Electron microscope
picture of a fly →



A SUMMARY OF DUALITY OF NATURE

Wave particle duality of physical objects



LIGHT

Wave nature -EM wave

Optical microscope

Interference

Particle nature -photons

Convert light to electric current

Photo-electric effect

PARTICLES

Wave nature

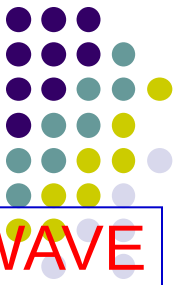
Matter waves -electron
microscope

Particle nature

Electric current
photon-electron collisions

Discrete (Quantum) states of confined
systems, such as atoms.

QUNATUM MECHANICS:



ALL PHYSICAL OBJECTS exhibit both PARTICLE AND WAVE LIKE PROPERTIES. THIS WAS THE STARTING POINT OF QUANTUM MECHANICS DEVELOPED INDEPENDENTLY BY WERNER HEISENBERG AND ERWIN SCHRODINGER.

Particle properties of waves: Einstein relation:
Energy of photon = h (frequency of wave).

Wave properties of particles: de Broglie relation:
wave length = $h/(\text{mass times velocity})$

Physical object described by a mathematical function called the wave function.

Experiments measure the Probability of observing the object.

A localized wave or wave packet:

A moving particle in quantum theory

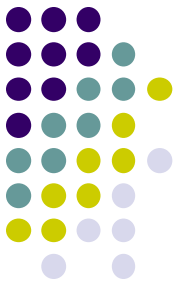


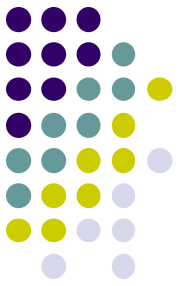
Spread in position

Spread in momentum

Superposition of waves
of different wavelengths
to make a packet

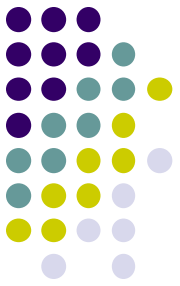
Narrower the packet , more the spread in momentum
Basis of Uncertainty Principle





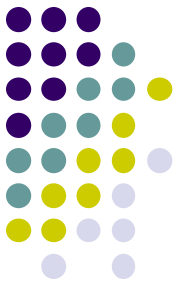
de Broglie concept of matter waves: dual nature of matter

Matter waves : de-Broglie concept



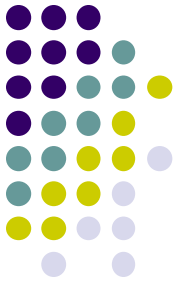
In 1924, Lewis de-Broglie proposed that matter has dual characteristic just like radiation. His concept about the dual nature of matter was based on the following observations:-

- (a) The whole universe is composed of matter and electromagnetic radiations. Since both are forms of energy so can be transformed into each other.
- (b) The matter loves symmetry. As the radiation has dual nature, matter should also possess dual character.



- According to the de Broglie concept of matter waves, the matter has dual nature.
- It means when the matter is moving it shows the wave properties (like interference, diffraction etc.) are associated with it and when it is in the state of rest then it shows particle properties.
- Thus the matter has dual nature. The waves associated with moving particles are matter waves or de-Broglie waves.

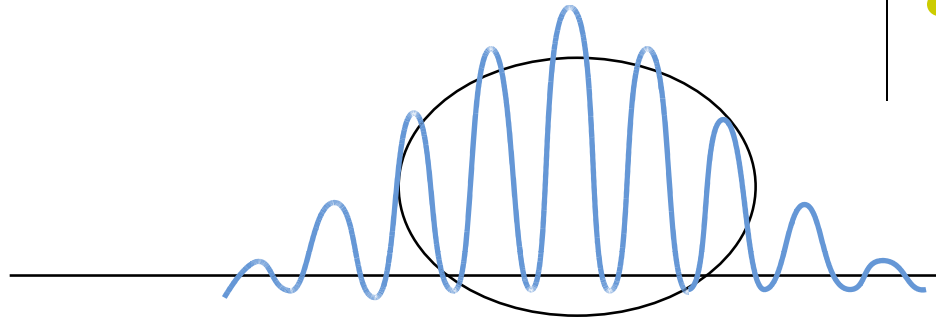
TRUE / FALSE



1. The photoelectric effect was used to show that light was composed of packets of energy proportional to its frequency.
2. The number of photons present in a beam of light is simply the intensity I divided by the photon energy $h\nu$.
3. Infrared light at a wavelength of 1.24 microns has photon energy of 1.5 eV.

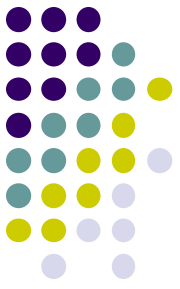
So is Light a Wave or a Particle ?

*Light is always both
Wave and Particle !*



On macroscopic scales, **large number of photons** look like they exhibit only wave phenomena.

A **single photon** is still a wave, but your act of trying to measure it makes it look like a localized particle.



Do Photons Have Momentum ?

What is momentum ?

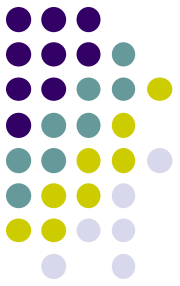
$$E = \frac{1}{2}mv^2 = \frac{1}{2}(mv) \cdot v = \frac{1}{2}p \cdot v$$

Just like Energy,
TOTAL MOMENTUM IS ALWAYS CONSERVED

Photons have energy and a finite velocity so there must be some momentum associated with photons !

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

Photon Momentum



IN FREE SPACE:

$$E = cp \Rightarrow p = \frac{E}{c} = \frac{\hbar\omega}{c} = \hbar k$$

IN OPTICAL MATERIALS:

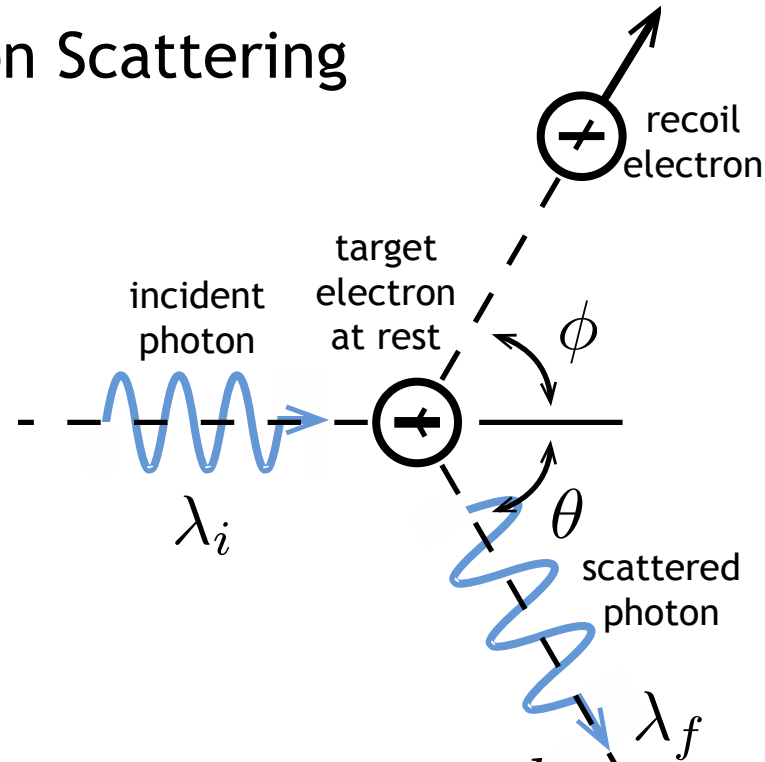
$$E = v_p p \Rightarrow p = \frac{E}{v_p} = \frac{\hbar\omega}{v_p} = \hbar k_{vac} n$$



Image by GFHund <http://commons.wikimedia.org/wiki/File:Compton, Arthur 1929 Chicago.jpg>
Wikimedia Commons.

In 1924, A. H. Compton performed an experiment where **X-rays** impinged on matter, and he measured the scattered radiation.

Compton Scattering

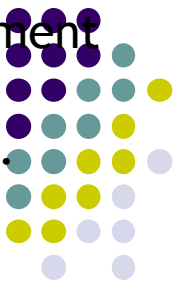


It was found that the scattered X-ray did not have the same wavelength !

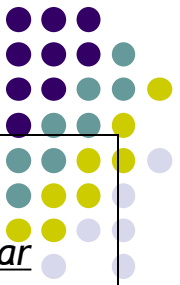
$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_o c} (1 - \cos \theta)$$

Compton found that if you treat the photons as if they were particles of zero mass, with energy $E = hc/\lambda$ and momentum $p = h/\lambda$.

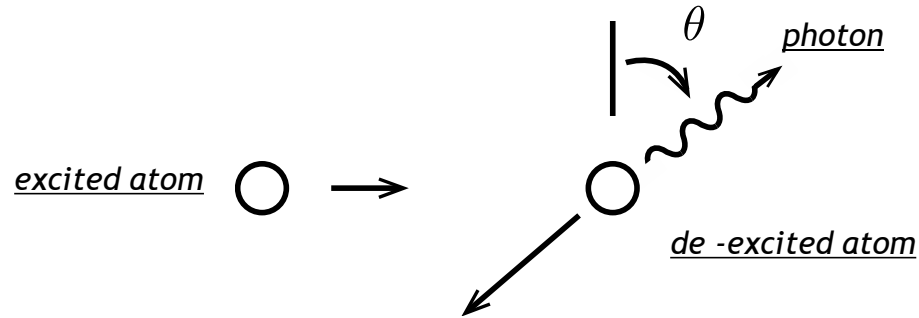
the collision behaves **just as if it were two billiard balls colliding !**
(with total momentum always conserved)



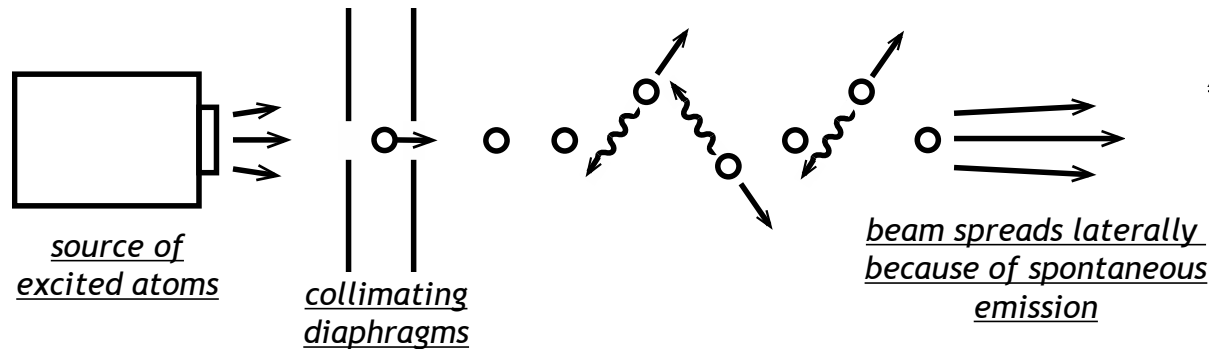
Manifestation of the Photon Momentum



SOURCE EMITTING A PHOTON

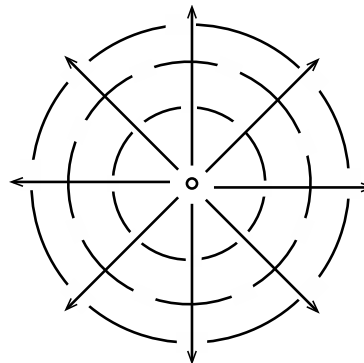


Conservation of linear momentum implies that an atom recoils when it undergoes spontaneous emission. The direction of photon emission (and atomic recoil) is not predictable.



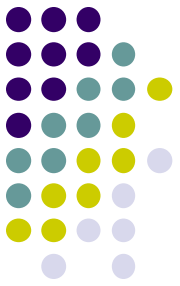
A well-collimated atomic beam of excited atoms will spread laterally because of the recoil associated with spontaneous emission.

SOURCE EMITTING AN EM WAVE



A source emitting a spherical wave cannot recoil, because the spherical symmetry of the wave prevents it from carrying any linear momentum from the source.

WAVELENGTH OF DE-BROGLIE WAVES



Consider a photon whose energy is given by

$$E = h\nu = hc/\lambda \quad (1)$$

If a photon possesses mass (rest mass is zero), then according to the theory of relativity, its energy is given by

$$E = mc^2 \quad (2)$$

From (1) and (2), we have

Mass of photon $m = h/c\lambda$

Therefore Momentum of photon

$$P = mc = hc/c\lambda = h/\lambda \quad (3)$$

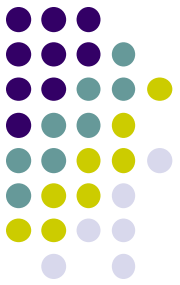
Or $\lambda = h/p$

If instead of a photon, we consider a material particle of mass m moving with velocity v , then the momentum of the particle, $p = mv$. Therefore, the wavelength of the wave associated with this moving particle is given by: h/mv —

Or $\lambda = h/p$ (But here $p = mv$) (4)

This wavelength is called De-Broglie wavelength.

Special Cases:



1. de-Broglie wavelength for material particle:

If E is the kinetic energy of the material particle of mass m moving with velocity v , then

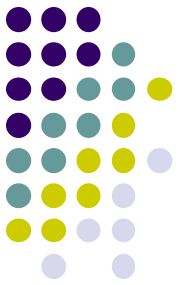
$$E = \frac{1}{2} mv^2 = \frac{1}{2} m^2 v^2 = \frac{p^2}{2m}$$

Or $p = \sqrt{2mE}$

Therefore by putting above equation in equation (4), we get de-Broglie wavelength equation for material particle as:

$$\lambda = h / \sqrt{2mE} \quad \text{--- (5)}$$

This is the de-Broglie wavelength for a material particle:



Special Cases:

2. dE-Broglie wavelength for particle in gaseous state:

According to kinetic theory of gases , the average kinetic energy of the material particle is given by

$$E = (3/2) kT$$

Where $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann's constant and T is the absolute temperature of the particle.

Also $E = p^2/2m$

Comparing above two equations, we get:

$$p^2/2m = (3/2) kT$$

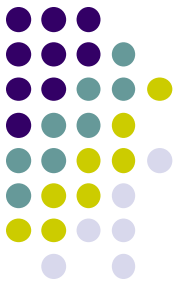
or $p = \sqrt{3mKT}$

Therefore Equation (4) becomes

$$\lambda = h/\sqrt{3mKT}$$

This is the de-Broglie wavelength for particle in gaseous state:

Special Cases:



3. de-Broglie wavelength for an accelerated electron:

Suppose an electron accelerates through a potential difference of V volt. The work done by electric field on the electron appears as the gain in its kinetic energy

That is $E = eV$

Also $E = p^2/2m$

Where e is the charge on the electron, m is the mass of electron and v is the velocity of electron, then

Comparing above two equations, we get:

$$eV = p^2/2m$$

$$\text{or } p = \sqrt{2meV}$$

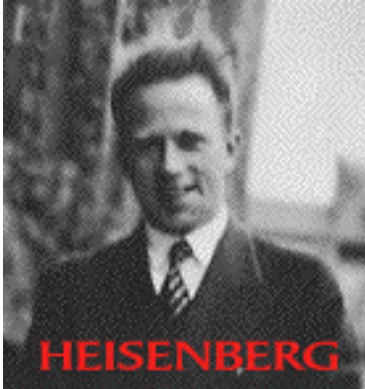
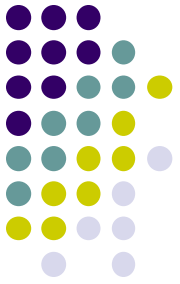
Thus by putting this equation in equation (4), we get the de-Broglie wavelength of the electron as

$$\lambda = h/\sqrt{2meV} = 6.63 \times 10^{-34} / \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}$$

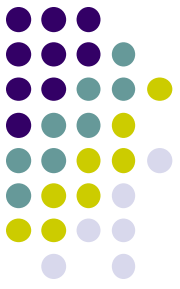
$$\lambda = 12.27/\sqrt{V} \text{ \AA}$$

This is the de-Broglie wavelength for electron moving in a potential difference of V volt.

ILLUSTRATION OF MEASUREMENT OF ELECTRON POSITION



Act of measurement
influences the electron
-gives it a kick and it
is no longer where it
was ! Essence of uncertainty
principle.



Classical world is Deterministic:

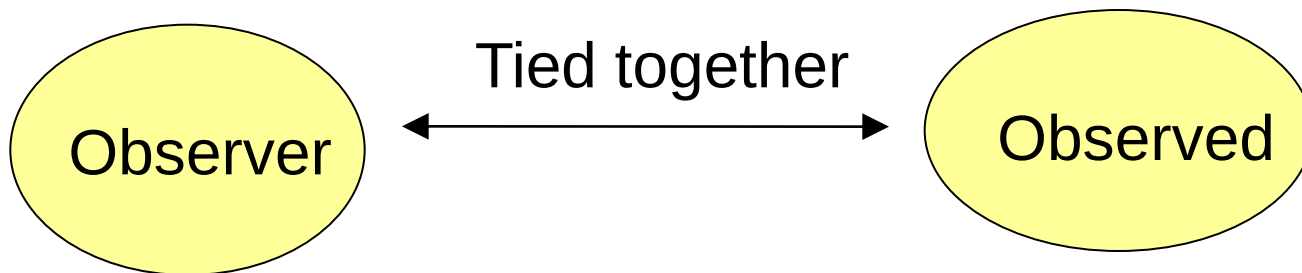
Knowing the position and velocity of all objects at a particular time

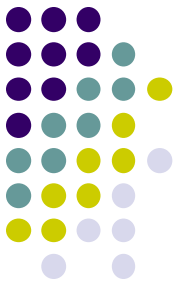
Future can be predicted using known laws of force and Newton's laws of motion.

Quantum World is Probabilistic:

Impossible to know position and velocity with certainty at a given time.

Only probability of future state can be predicted using known laws of force and equations of quantum mechanics.





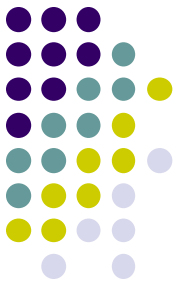
Before observation it is impossible to say
Whether an object is a wave or a particle
Or whether it exists at all !!

Quantum mechanics is a probabilistic theory of nature

Uncertainty relations of heisenberg allow you to
Get away with anything provided you do it fast
Enough !! example: bank employee withdrawing cash, using it
,but Replacing it before he can be caught ...

Confined physical systems – an atom – can only
Exist in certain allowed states

they are quantized



Common sense view of the world is
an Approximation of the underlying
basic Quantum description of our
physical World !

In the Copenhagen interpretation of
Bohr and Heisenberg it is impossible
in Principle for our world to be
Deterministic !

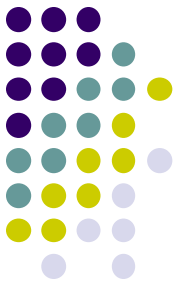
Einstein, a founder of QM was
Uncomfortable with this
Interpretation

God does not play dice !

Bohr and Einstein in discussion 1933



Heisenberg realized that ...



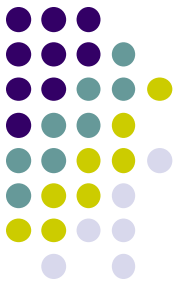
- In the world of very small particles, one cannot measure any property of a particle without interacting with it in some way
- This introduces an unavoidable uncertainty into the result
- One can never measure all the properties exactly



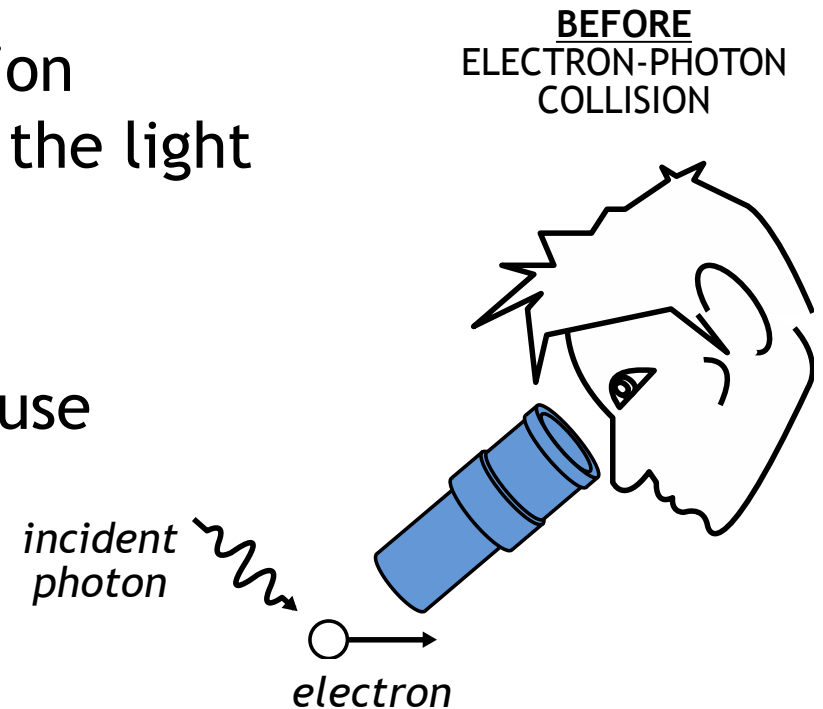
Werner Heisenberg (1901-1976)

Image in the Public Domain

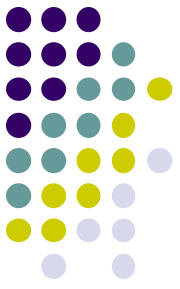
Measuring Position and Momentum of an Electron



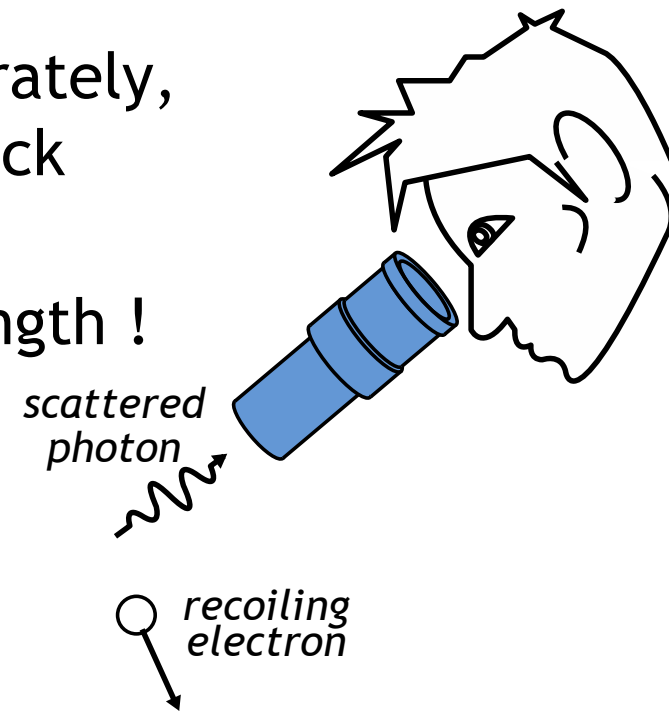
- Shine light on electron and detect reflected light using a microscope
- Minimum uncertainty in position is given by the wavelength of the light
- So to determine the position accurately, it is necessary to use light with a short wavelength



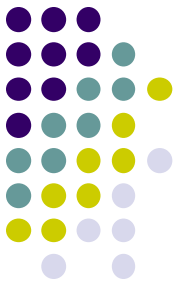
Measuring Position and Momentum of an Electron



- By Planck's law $E = hc/\lambda$, a photon with a short wavelength has a large energy
- Thus, it would impart a large 'kick' to the electron
- But to determine its momentum accurately, electron must only be given a small kick
- This means using light of long wavelength !

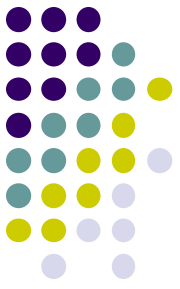


Implications



- It is impossible to know *both* the position and momentum exactly, i.e., $\Delta x=0$ and $\Delta p=0$
- These uncertainties are inherent in the physical world and have nothing to do with the skill of the observer
- Because h is so small, these uncertainties are not observable in normal everyday situations

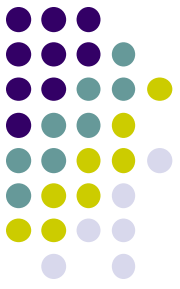
$$\hbar = 1.054 \times 10^{-34} [\text{J} \cdot \text{s}]$$



Example of Baseball

- A pitcher throws a 0.1-kg baseball at 40 m/s
- So momentum is $0.1 \times 40 = 4 \text{ kg m/s}$
- Suppose the momentum is measured to an accuracy of 1 percent , i.e.,

$$\Delta p = 0.01 p = 4 \times 10^{-2} \text{ kg m/s}$$

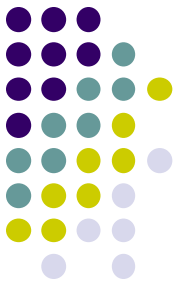


Example of Baseball (cont'd)

- The uncertainty in position is then

$$\Delta x \geq \frac{h}{4\pi\Delta p} = 1.3 \times 10^{-33} \text{m}$$

- No wonder one does not observe the effects of the uncertainty principle in everyday life!

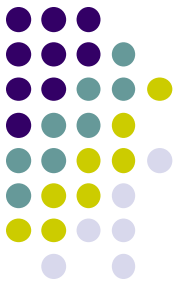


Example of Electron

- Same situation, but baseball replaced by an electron which has mass 9.11×10^{-31} kg traveling at 40 m/s
- So momentum = 3.6×10^{-29} kg m/s
and its uncertainty = 3.6×10^{-31} kg m/s
- The uncertainty in position is then

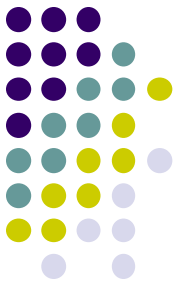
$$\Delta x \geq \frac{h}{4\pi\Delta p} = 1.4 \times 10^{-4} \text{ m}$$

Classical World

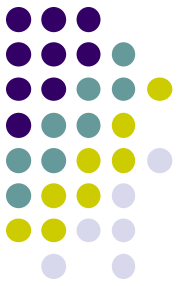


- The observer is objective and passive
- Physical events happen independently of whether there is an observer or not
- This is known as **objective reality**

Role of an Observer in Quantum Mechanics



- The observer is *not* objective and passive
- The act of observation changes the physical system irrevocably
- This is known as **subjective reality**



One might ask:

*“If light can behave like a particle,
might particles act like waves”?*

YES !

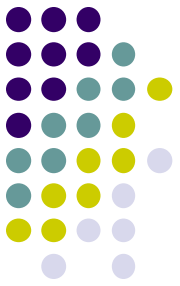
Particles, like photons, also have a wavelength given by:

$$\lambda = h/p = h/mv$$

The wavelength of a particle depends on its momentum,
just like a photon!

The main difference is that matter particles have mass,
and photons don't!

Matter Waves



Compute the wavelength of a 10 [g] bullet moving at 1000 [m/s].

$$\begin{aligned}\lambda &= h/mv = 6.6 \times 10^{-34} \text{ [J s]} / (0.01 \text{ [kg]})(1000 \text{ [m/s]}) \\ &= 6.6 \times 10^{-35} \text{ [m]}\end{aligned}$$

This is **immeasurably small**

For ordinary “everyday objects,”
we don’t experience that

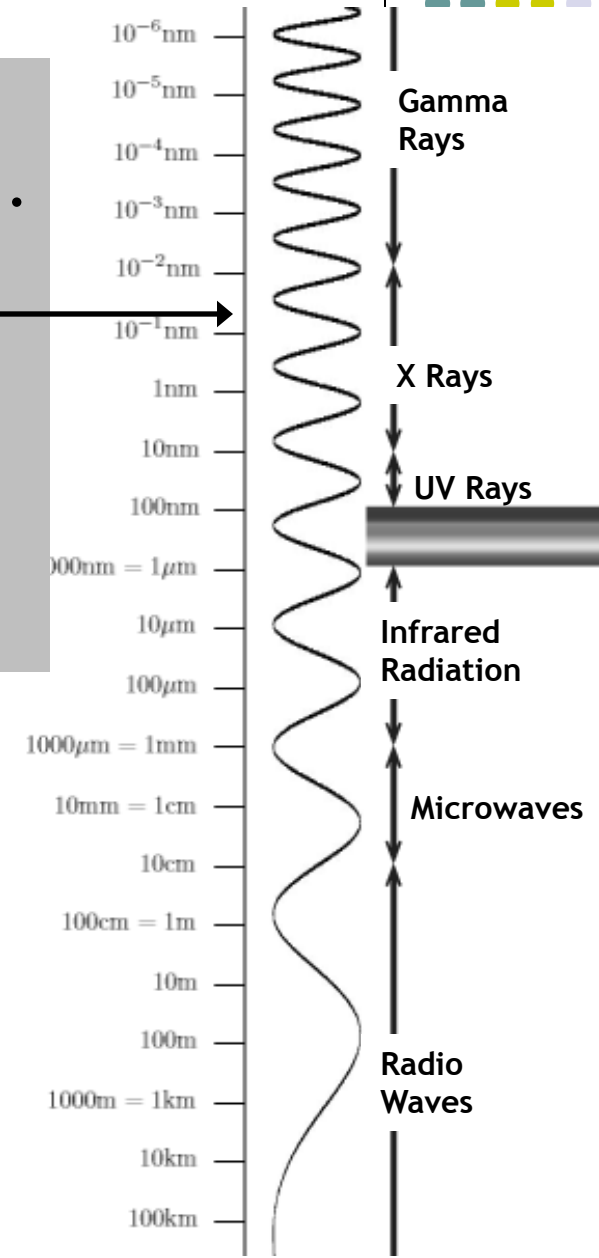
MATTER CAN BEHAVE AS A WAVE

But, what about small particles ?

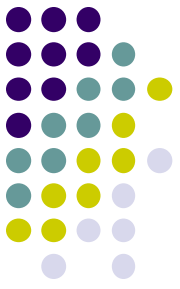
Compute the wavelength of an electron ($m = 9.1 \times 10^{-31}$ [kg]) moving at 1×10^7 [m/s].

$$\begin{aligned}\lambda &= h/mv \\ &= 6.6 \times 10^{-34} \text{ [J s]} / (9.1 \times 10^{-31} \text{ [kg]})(1 \times 10^7 \text{ [m/s]}) \\ &= 7.3 \times 10^{-11} \text{ [m]}. \\ &= 0.073 \text{ [nm]}\end{aligned}$$

These electrons
have a wavelength in the region
of X-rays



Wavelength versus Size



With a visible light microscope, we are limited to being able to resolve objects which are at least about $0.5 \times 10^{-6} \text{ m} = 0.5 \mu\text{m} = 500 \text{ nm}$ in size.

This is because visible light, with a wavelength of $\sim 500 \text{ nm}$ cannot resolve objects whose size is smaller than it's wavelength.

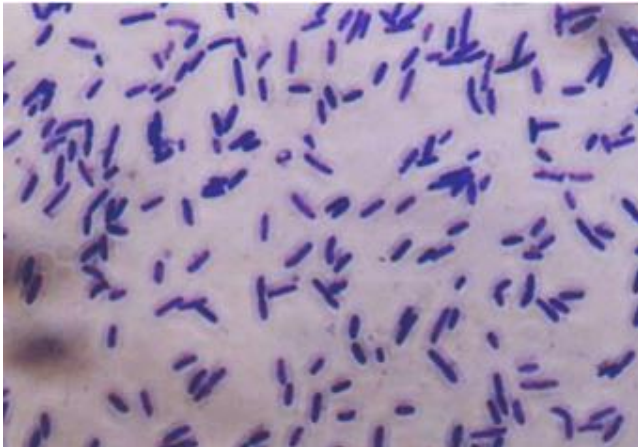


Image is in the public domain

Bacteria, as viewed
using visible light

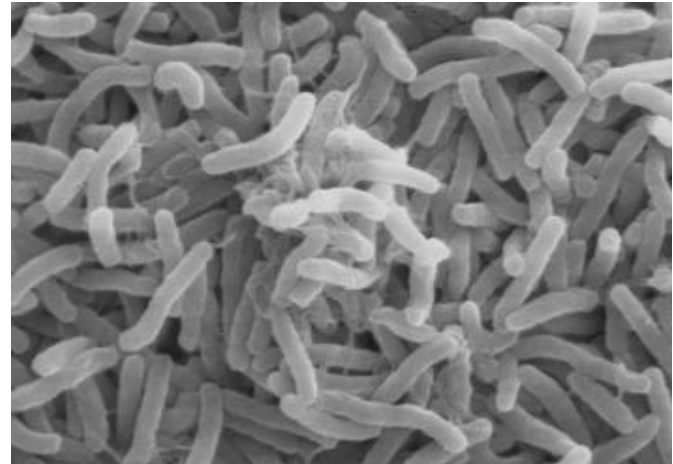
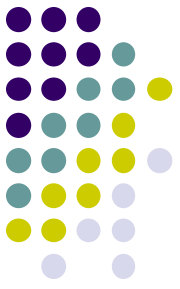


Image is in the public domain

Bacteria, as viewed
using electrons!

Electron Microscope



The electron microscope is a device which uses the wave behavior of electrons to make images which are otherwise too small for visible light!

This image was taken with a Scanning Electron Microscope (SEM).

SEM can resolve features as small as 5 nm. This is about 100 times better than can be done with visible light microscopes!

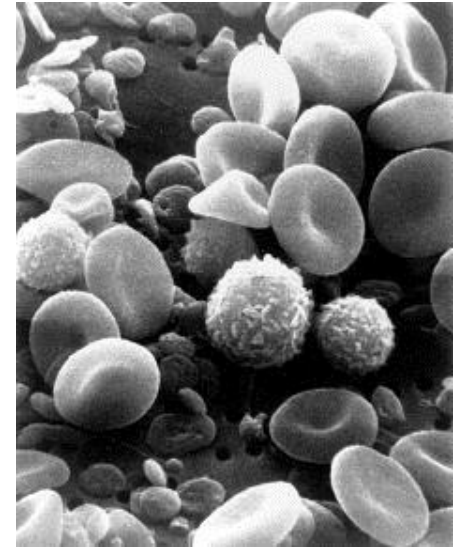
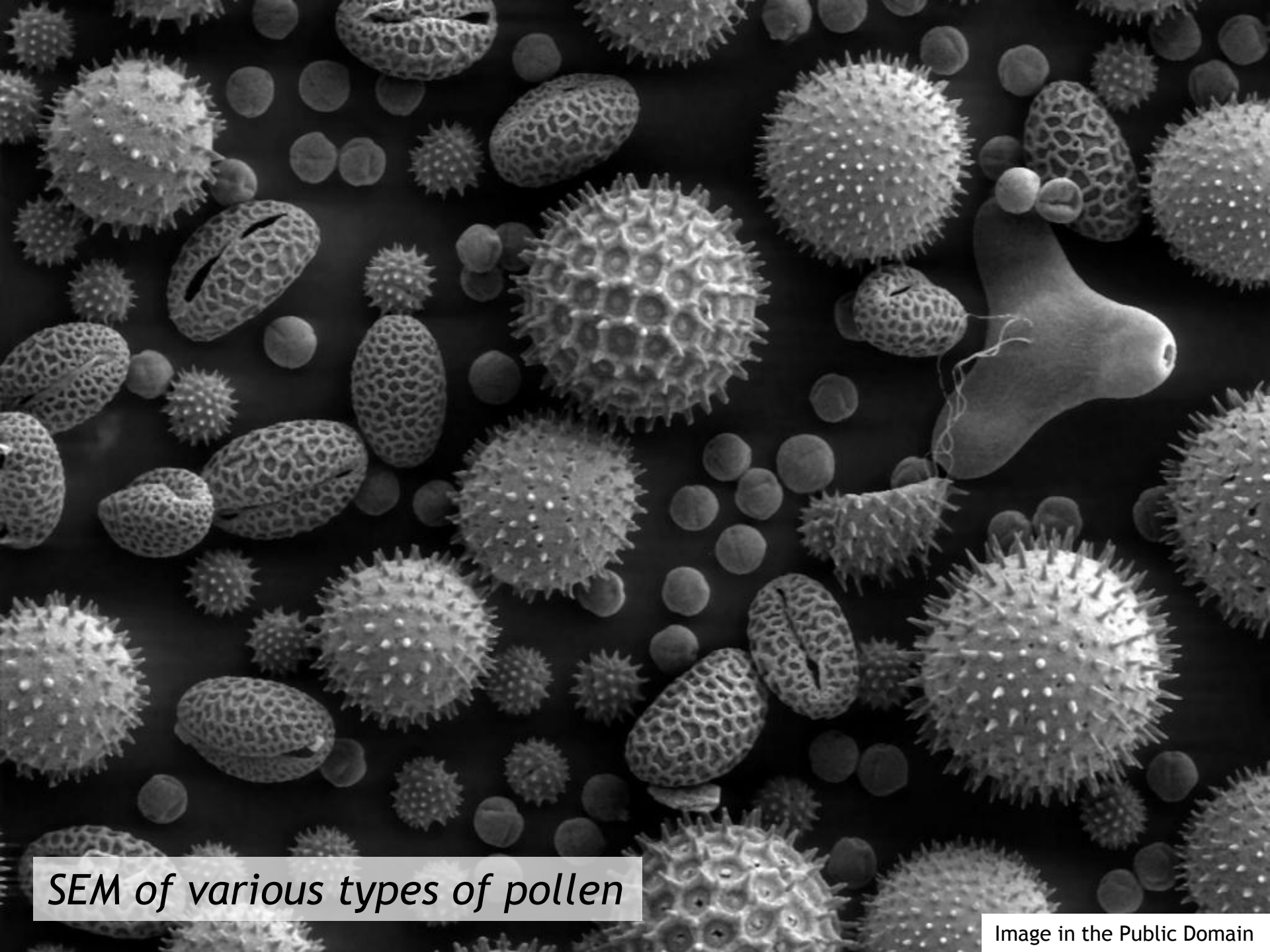


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IMPORTANT POINT:

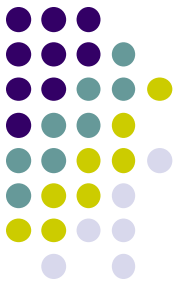
High energy particles can be used to reveal the structure of matter !



SEM of various types of pollen



SEM of an ant head



Summary

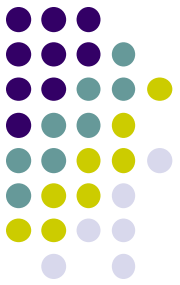
- ❑ Light is made up of photons, but in macroscopic situations it is often fine to treat it as a wave.

- ❑ Photons carry both energy & momentum.

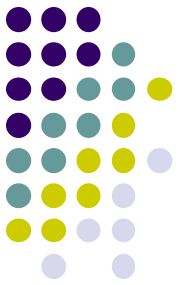
$$E = hc/\lambda \quad p = E/c = h/\lambda$$

- ❑ Matter also exhibits wave properties. For an object of mass m , and velocity, v , the object has a wavelength, $\lambda = h / mv$
- ❑ One can probe ‘see’ the fine details of matter by using high energy particles (they have a small wavelength !)

Heisenberg uncertainty principle



Statement: According to Heisenberg uncertainty principle, it is impossible to measure the exact position and momentum of a particle simultaneously within the wave packet.



We know, group velocity of the wave packet is given by

$$v_g = \Delta\omega / \Delta k$$

Where ω is the angular frequency and k is the propagation constant or wave number

But v_g is equal to the particle velocity v

$$\text{Thus } v_g = v = \Delta\omega / \Delta k \quad (1)$$

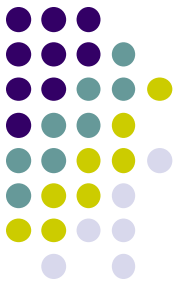
$$\text{But } \omega = 2\pi f$$

Where f is the frequency

$$\text{Therefore } \Delta\omega = 2\pi \Delta f \quad (2)$$

$$\text{Also } k = 2\pi / \lambda$$

$$\text{Since de-Broglie wavelength } \lambda = h/p$$



By putting this value in equation of k , we get

$$k = 2\pi p / \lambda$$

Therefore
$$\Delta k = 2\pi \Delta p / \lambda \quad (3)$$

Put equations (2) and (3) in equation (1), we get

$$v = 2\pi h \Delta f / 2\pi \Delta p = h \Delta f / \Delta p \quad (4)$$

Let the particle covers distance Δx in time Δt , then particle velocity is given by

$$v = \Delta x / \Delta t \quad (5)$$

Compare equations (4) and (5), we get

$$\Delta x / \Delta t = h \Delta f / \Delta p$$

Or
$$\Delta x \cdot \Delta p = h \Delta f \Delta t \quad (6)$$

The frequency Δf is related to Δt by relation

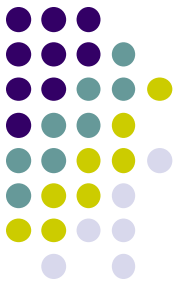
$$\Delta t \geq 1 / \Delta f \quad (7)$$

Hence equations (6) becomes

$$\Delta x \cdot \Delta p \geq h$$

A more sophisticated derivation of Heisenberg's uncertainty principle gives

$$\Delta x \cdot \Delta p = h / 2\pi \quad (8)$$

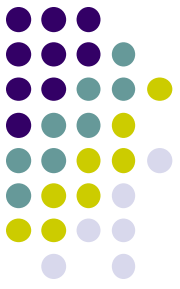


Which is the expression of the **Heisenberg uncertainty principle**.

As the particle is moving along x-axis. Therefore, the momentum in equation (8) of Heisenberg's uncertainty principle should be the component of the momentum in the x-direction, thus equation Heisenberg's uncertainty principle can be written as,

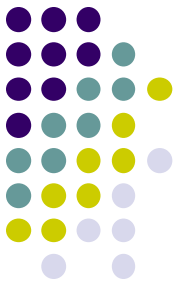
$$\Delta x \cdot \Delta p_x = h/2\pi \quad (9)$$

Note: There can not be any uncertainty if momentum is along y direction.



- The Heisenberg uncertainty principle based on quantum physics explains a number of facts which could not be explained by classical physics.
- One of the applications is to prove that electron can not exist inside the nucleus. It is as follows:-

Non-existence of electrons in the nucleus



In this article, we will prove that electrons cannot exist inside the nucleus.

But to prove it, let us assume that electrons exist in the nucleus. As the radius of the nucleus is approximately 10^{-14} m. If electron is to exist inside the nucleus, then uncertainty in the position of the electron is given by

$$\Delta x = 10^{-14} \text{ m}$$

According to uncertainty principle,

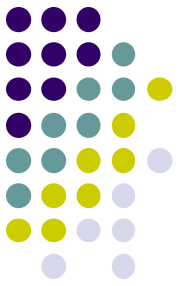
$$\Delta x \Delta p_x = h/2\pi$$

Thus
$$\Delta p_x = h/2\pi \Delta x$$

Or
$$\Delta p_x = 6.62 \times 10^{-34} / 2 \times 3.14 \times 10^{-14}$$

$$\Delta p_x = 1.05 \times 10^{-20} \text{ kg m/ sec}$$

If this is p the uncertainty in the momentum of electron, then the momentum of electron should be at least of this order, that is $p = 1.05 \times 10^{-20} \text{ kg m/sec}$.



An electron having this much high momentum must have a velocity comparable to the velocity of light. Thus, its energy should be calculated by the following relativistic formula

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$$E = \sqrt{(9.1 \times 10^{-31})^2 (3 \times 10^8)^4 + (1.05 \times 10^{-20})^2 (3 \times 10^8)^2}$$

$$= \sqrt{(6707.61 \times 10^{-30}) + (9.92 \times 10^{-24})}$$

$$= (0.006707 \times 10^{-24}) + (9.92 \times 10^{-24})$$

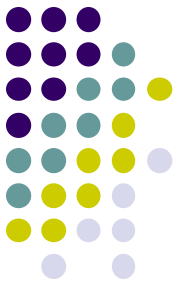
$$= \sqrt{9.9267 \times 10^{-24}}$$

$$E = 3.15 \times 10^{-12} \text{ J}$$

$$\text{Or} \quad E = 3.15 \times 10^{-12} / 1.6 \times 10^{-19} \text{ eV}$$

$$E = 19.6 \times 10^6 \text{ eV}$$

$$\text{Or} \quad E = 19.6 \text{ MeV}$$



- Therefore, if the electron exists in the nucleus, it should have an energy of the order of 19.6 MeV. However, it is observed that beta-particles (electrons) ejected from the nucleus during β -decay have energies of approximately 3 MeV, which is quite different from the calculated value of 19.6 MeV. Second reason that electron can not exist inside the nucleus is that experimental results show that no electron or particle in the atom possess energy greater than 4 MeV.
- Therefore, it is confirmed that electrons do not exist inside the nucleus.