Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

PROGRAMME: B.Sc. PHYSICS HONOURS

SEMESTER - VI

Course: Electromagnetic Theory (CC-XIII)

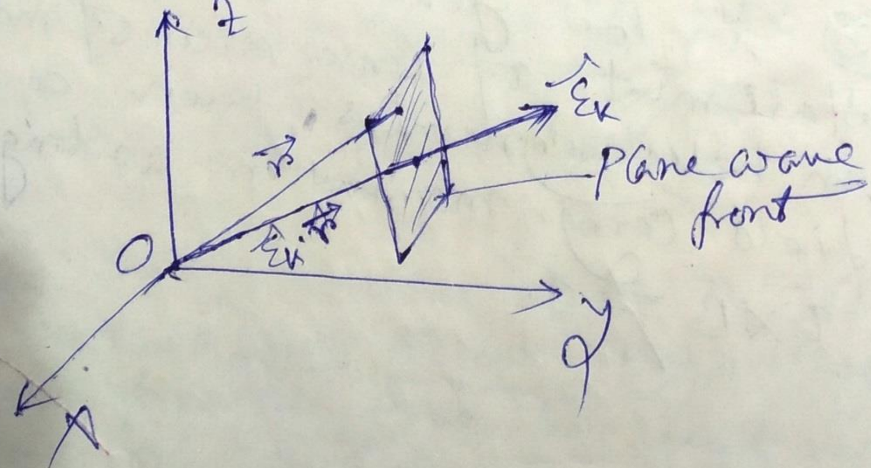
Lecture Note: 5

Propagation of Plane Electromagnetic Wave in dielectrics

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

Plane Electromagnetic Wave in a dielectric :-

An e.m. wave is said to be plane wave in which the field vector $\vec{E}$ or $\vec{B}$ or its components and amplitude are same over all the points on a plane normal to the direction of propagation. Also the state of vibration (Phase) at all points on a plane at any time t are same. The plane is actually wave front which advances with a velocity $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ normal to ~~plane~~ plane. The field vectors or components that lie on this plane are function of perpendicular distance $\hat{\epsilon}_k \cdot \vec{r}$ of that plane from the origin and also the function of time.



Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

The mathematical equation of plane em wave are represented by

$$\left.\begin{aligned} \nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \\ \nabla^2\vec{B} - \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} = 0 \end{aligned}\right\} \text{--- (1)}$$

These wave travell in space in any arbitrary direction, determined by the unit propagation vector $\hat{\varepsilon}_k = \vec{k}/|\vec{k}|$, where $|\vec{k}| = 2\pi/\lambda$.

The plane wave solution of these two equations in complex form are

$$\left.\begin{aligned} \vec{E}(\vec{r},t) = \hat{\varepsilon}_1 E_0 e^{j(\vec{k}\cdot\vec{r} - \omega t)} \\ \vec{B}(\vec{r},t) = \hat{\varepsilon}_2 B_0 e^{j(\vec{k}\cdot\vec{r} - \omega t)} \end{aligned}\right\} \text{--- (2)}$$

where, E_0 and B_0 are complex amplitude of $\vec{E}$ and $\vec{B}$ wave. $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ are the unit vectors indicating the direction of vibration of $\vec{E}$ and $\vec{B}$ wave, called polarization vectors.

The actual fields are the real part of these equation

$$\left.\begin{aligned} E = E_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \delta_1) \\ B = B_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \delta_2) \end{aligned}\right\} \text{--- (3)}$$

where δ_1 and δ_2 are the starting phase of $\vec{E}$ and $\vec{B}$ wave respectively.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

The ratio $\frac{\omega}{k} = c$, or v (in medium)

The wave equation must satisfy the Maxwell's equation of em wave. So from 1st Maxwell equation

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\Rightarrow \nabla\cdot\left\{\hat{\varepsilon}_1 E_0\, e^{+j(\vec{k}\cdot\vec{r}-\omega t)}\right\} = 0$$

$$\Rightarrow j\vec{k}\cdot\hat{\varepsilon}_1 E_0\, e^{j(\vec{k}\cdot\vec{r}-\omega t)} = 0 \quad \text{(4)}$$

[Here $j = \sqrt{-1}$

in Complex, $\frac{\partial}{\partial x} \equiv jk_x$, $\frac{\partial}{\partial y} \equiv jk_y$

$\frac{\partial}{\partial z} = jk_z$.

So the operator $\nabla \equiv j[\hat{i}k_x + \hat{j}k_y + \hat{k}k_z]$

$= j\vec{k}$.]

or from eq^n (4) we get

$$\vec{k}\cdot\vec{E} = 0 \quad \text{(4)}$$

i.e $\vec{E}$ is perpendicular to $\vec{k}$ i.e $\vec{E}$ is transverse wave.

Similarly from Maxwell 2nd eq^n, we get, $\vec{k}\cdot\vec{B} = 0$ ——— (5)

So $\vec{B}$ is also perpendicular to $\vec{k}$, the propagation vector.

From Maxwell 3rd eq^n

$$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

In wave motion the operator

$$\nabla \equiv j\vec{K} \text{ and } \frac{\partial}{\partial t} \equiv -j\omega, \quad j = \sqrt{-1}$$

So

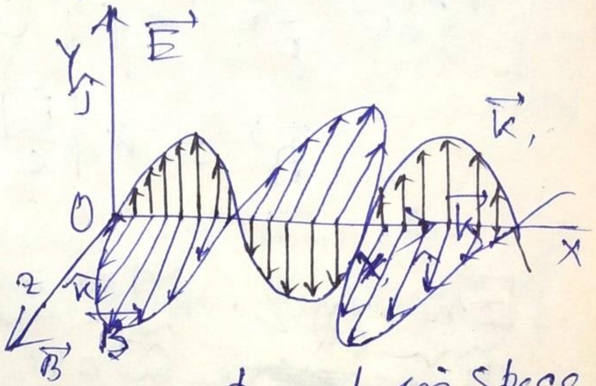
$$j\vec{K} \times \vec{E} = +j\omega\vec{B}$$

$$\Rightarrow \vec{B} = \frac{\vec{K} \times \vec{E}}{\omega} \quad \text{——— (6)}$$

$$\Rightarrow \vec{E} \cdot \vec{B} = 0$$

So $\vec{E}$ is also perpendicular to $\vec{B}$.

So $\vec{K}$, $\vec{E}$, $\vec{B}$ is space form a orthogonal system which are mutually perpendicular. It is shown in fig.



So $\vec{E}$ and $\vec{B}$ wave travel in space as a transverse em. wave in same speed of light $c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8$ m/s and also in same phase.

From eqn (6), the magnetic field

$$\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$$

or $$|\vec{B}| = \frac{K|\vec{E}|}{\omega} = |\vec{E}|/c.$$

or $$|B_0| = |E_0|/c, \text{ where } c = \omega/K$$ velocity of light.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

This gives that the amplitude of magnetic wave is very weak compared to the amplitude of electric wave.

So the physical fields are

$$\vec{E} = \hat{\varepsilon}_1 |E_0| \cos(\vec{k}\cdot\vec{r} - \omega t + \delta).$$

$$\vec{B} = \hat{\varepsilon}_{12} |E_0|/c \cos(\vec{k}\cdot\vec{r} - \omega t + \delta).$$

For a given direction of propagation say X are

$$\vec{E} = \hat{j}|E_0| \cos(kx - \omega t + \delta)$$

$$\vec{B} = \hat{k} \left|\frac{E_0}{c}\right| \cos(kx - \omega t + \delta)$$

Energy & Momentum of a Plane em. ~~Progressive~~ Wave :-

The energy density stored in an em wave in vacuum is

$$u_{em} = \frac{1}{2}(\epsilon_0 E^2 + B^2/\mu_0) \ J/m^3.$$

As the frequency of $\vec{E}$ and $\vec{B}$ wave is very high ($\simeq 10^{10} - 10^{15}$ Hz), we have to take average value of em energy density

$$\overline{u_{em}} = \frac{1}{2}\epsilon_0 \overline{E^2} + \frac{1}{2}\frac{\overline{B^2}}{\mu_0}.$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

$$= \frac{1}{2}\left[\epsilon_0 \frac{\bar{E}\cdot\bar{E}^*}{2} + \frac{1}{\mu_0}\frac{\bar{B}\cdot\bar{B}^*}{2}\right]$$

Now $\bar{E} = \hat{\epsilon}_1 E_0 e^{j(k\cdot r - \omega t)}$

$\bar{B} = \hat{\epsilon}_2 \frac{E_0}{c} e^{j(k\cdot r - \omega t)}$

$\therefore E^* = \hat{\epsilon}_1 E_0^* e^{-j(k\cdot r - \omega t)}$

$\bar{B}^* = \hat{\epsilon}_2 \frac{E_0^*}{c} e^{-j(k\cdot r - \omega t)}$.

$$\therefore \bar{u}_{em} = \frac{1}{2}\left[\epsilon_0 \frac{1}{2}\cdot |E_0|^2 + \frac{1}{2\mu_0}|B|^2\right]$$

$$= \frac{1}{4}\left[\epsilon|E_0|^2 + \frac{1}{\mu_0}\cdot\frac{|E_0|^2}{c^2}\right]$$

$$= \frac{1}{4}\left[2\epsilon_0|E_0|^2\right] \quad \left[\because c^2 = \frac{1}{\mu_0\epsilon_0}\right]$$

$$\bar{u}_{em} = \frac{1}{2}\epsilon_0|E_0|^2.$$

This equation shows is $\bar{u}_{em}$ is directly proportional to square of amplitude.

ii) $\bar{u}_e$ and $\vec{u}_m$ is same i.e the total energy ~~carried~~ is shared equally by $\bar{E}$ and $\bar{B}$ field although the amplitude of $\vec{B}$ field very negligible compared to $\vec{E}$ field.

For a plane em wave

$$\vec{B} = \frac{\bar{K}\times\bar{E}}{\omega}.$$

$$\therefore \bar{B}^* = \frac{\bar{K}\times\bar{E}^*}{\omega}.$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

The pointing vector of em wave

$$\bar{S} = \frac{\bar{E} \times \bar{B}}{\mu_0},$$

Its magnitude gives the energy crossing through unit area per second normally. Its direction is along $\hat{\varepsilon}_k$, the direction of propagation.

The time average of pointing vector

$$\langle \vec{S} \rangle = \left\langle \frac{EB \cancel{\sin\theta}}{\mu_0} \right\rangle \hat{\varepsilon}_k$$

$$= \left\langle \frac{EB}{\mu_0} \right\rangle \hat{\varepsilon}_k$$

Now $EB = E_0 B_0 \cos^2(k \cdot r - \omega t)$

$$\therefore \langle EB \rangle = |E_0||B_0| \times \frac{1}{2}.$$

$$\therefore \langle \vec{S} \rangle = \frac{|E_0||B_0|}{2\mu_0} \hat{\varepsilon}_k$$

$$= \frac{1}{2} \frac{E_0^2}{\mu_0 c} \cdot \hat{\varepsilon}_k$$

$$= \frac{1}{2} \epsilon_0 c |E_0|^2 \hat{\varepsilon}_k$$

So the ~~avarag~~ average magnitude of $\vec{S}$

$$\bar{S} = \frac{1}{2} \epsilon_0 c |E_0|^2.$$

= average energy per sec per unit area normally.

= Intensity of em wave.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

So ~~the average~~
So the intensity of em wave is proportional to the square of amplitude

So the average momentum of em wave is

$$\bar{g}_{em} = \bar{S}/c^2$$
$$= \frac{1}{2}\frac{\epsilon_0 c |E_0|^2}{c^2}$$
$$= \frac{1}{2}\epsilon_0 |E_0|^2/c$$
$$\boxed{\bar{g}_{em} = \frac{\epsilon_0 |E_0|^2}{2c}} = \frac{\bar{u}_{em}}{c}$$

So the average momentum also proportional to square of amplitude of E wave.

A parallel plate capacitor with circular plates of radious $a = 4$ cm, and separation $d = 2$ mm is shown in fig. This capacitor is being charged by a battery of emf $V = 40$ V, through a ~~res~~ resistor R. At a particular instant t, the charge on the capacitor is 1×10^{-10} C, while the current in the circuit 20 μA. Consider the cylindrical surface of radius 4 cm and ~~height~~ height 2 mm between the plates. Show that at the instant t the magnitude of pointing vector at any point on the curved surface of the cylinder is 0.358. What is the direction of N.? Show explicitly that the surface integral of N over this surface equals the rate at which

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

Radiation Pressure:-

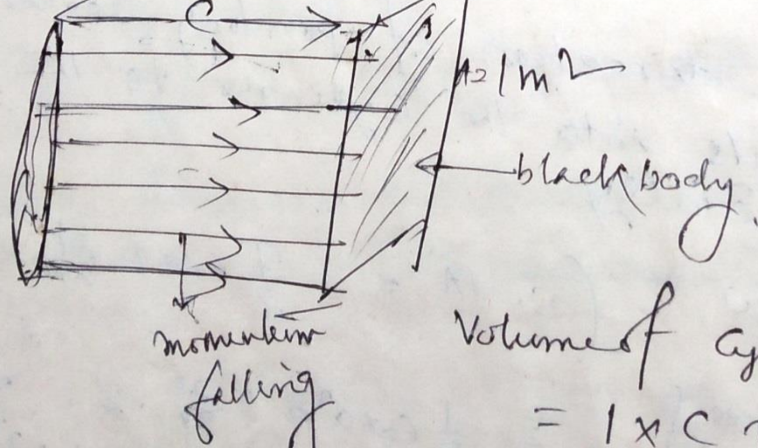
If the radiation momentum $\bar{g}_{em} = \frac{\bar{u}_{em}}{c}$ is ~~total~~ totally absorbed by a surface (black body) on which it falls, the momentum absorbed per sec per unit area normally is the radiation pressure. The momentum absorbed per sec per unit area is equal to the momentum contained in a cylinder of length 'c' (vacuum) and of ~~cut~~ unit cross-section. So the radiation pressure

$$= 1 \times c \times \vec{g}_{em}$$

$$= \cancel{c} \times \frac{\bar{u}_{em}}{\cancel{c}}$$

$$= \bar{u}_{em} = \frac{1}{2}\epsilon_0 |E_0|^2 \; N/m^2$$

$= $ average energy density



Volume of cylinder

$= 1 \times c \; m^3$

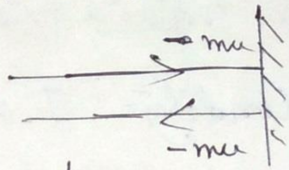
$= c \; m^3$

So force on black body

$=$ pressure $\times$ area.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

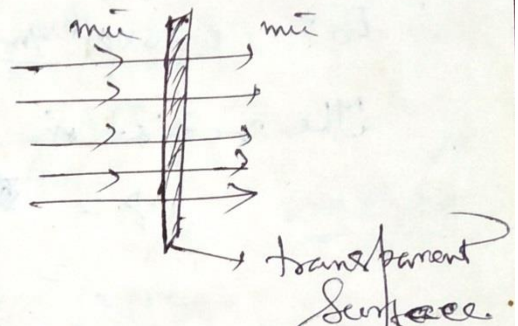
If the incident radiation is totally reflected by a polish surface (mirror) then ~~raditiat radi~~ radiation pressure will be doubled.



So change in momentum

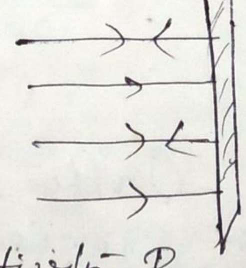
$$= mu - (-mu) = 2mu$$

~~For~~ On a ~~transferent~~ transparent surface no radiation ~~prem~~ pressure will be ~~applied~~ applied.



If ~~a~~ radiation fall on a surface of 50% reflectivity then radiation pressure will be

$$p = \frac{3}{2} \times \overrightarrow{u_{em}}.$$



In general, the radiation pressure of on a surface of reflectivity R

$$= (R+1)\,\overrightarrow{u_{em}}$$

$= \overrightarrow{u_{em}}$ for absorbtive ~~medium~~ surface $R=0$.

$= 2\,\overrightarrow{u_{em}}$ for fully reflecting ~~medium~~ $R=1$

$= \frac{3}{2}\,\overrightarrow{u_{em}}$. for 50% reflectivity $R=1/2$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

1. If the source has intensity $1\ KW/m^2$. Then calculate the electric field and radiation pressure.

Intensity $I = \bar{S} = \frac{1}{2}\epsilon_0 c|E_0|^2 = 10^3 W/m^2$

$|E_0|^2 = 2\bar{S}/\epsilon_0 c$

$= \frac{2\times 10^3}{8.85\times 10^{-12}\times 3\times 10^8}$

$= \frac{2\times 1}{3\times 8.85} 10^{+7}$

$\therefore E_0 = 0.86\times 10^3\ V/m$

or $E_0 = \sqrt{2\epsilon_0 \cdot 10^3}\ V/m$

$= \sqrt{2\times 10^3\times 8.85\times 10^{-12}}$

$= 10^{-5}\sqrt{2\times 8.85\times 10}\ V/m$

$E_0 = 13.3\times 10^{-5}\ V/m$

The radiation pressure

$P = c\times \bar{g}_{em}$

$= c\times \bar{S}/c^2 = \bar{S}/c$

$= \frac{10^3}{3\times 10^8}\ N/m^2$

$$\boxed{P = 0.33\times 10^{-5}\ N/m^2}$$

2. On the surface of earth we recieve intensity $1.33\ KW/m^2$ from the sun. Calculate the electric field associated with the sunlight on earth.

$I = \bar{S} = \frac{1}{2}\epsilon_0 c|E_0|^2$

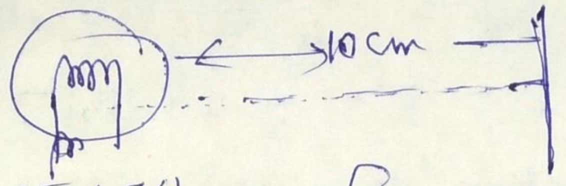
$\therefore E_0 = \sqrt{\frac{2\bar{S}}{\epsilon_0 c}} = \sqrt{\frac{2\times 1.33\times 10^3}{8.85\times 10^{-12}\times 3\times 10^8}}$

$= 0.316\times 10^3\sqrt{10} = 10^3\ V/m$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

Q. Find the poynting vector at a distance 10 cm from a 1000 watt Na lamp.

Soln



$\omega = 1000\,W$.

The intensity of the lamp at a distance r from source

$$I = \frac{\text{watt}}{4\pi r^2}$$

$$= \frac{1000}{4\pi\times(10\times10^{-2})^2}$$

$$= \frac{1000}{4\pi\times10^{-2}}$$

$$= 10^5/4\pi \text{ watt/m}^2$$

So $I = \bar{S} = \frac{1}{2}\epsilon_0 c|E_0|^2$.

$\therefore$ peak $|E_0|^2 = 2\bar{S}/\epsilon_0 c$.

The poynting vector's magnitude $= |S| = \frac{E_0^2}{\mu_0 c}$

$$= \frac{2\bar{S}}{\epsilon_0 c\,\mu_0 c}$$

$$= 2\bar{S}$$

$$= 2\times 10^5/4\pi \text{ Joule/m}^2$$

$$= 0.159\times10^5 \text{ J/m}^2$$

Q4. An 100 watt lamp is placed at a distance 1 m in front of a plane mirror with reflectivity 0.8. Calculate the radiation pressure.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

The intensity of the lamp at a dist 1m.

$$I = \frac{100}{4\pi 1^2} = 100/4\pi \ \text{watt}/m^2$$

$$\therefore \quad I = \bar{S} = \frac{1}{2}\epsilon_0 c |E_0|^2.$$

radiation pressure

$$p = (R+1)\,\vec{S}/c \quad N/m^2$$

$$= (0.8+1)\frac{100}{4\pi c}$$

$\boxed{R = \text{reflectivity} = 0.8}$

$$= 1.8 \times \frac{100}{4\pi \times 3\times 10^8} \ N/m^2$$

$$= 0.0477 \times 10^{-6}$$

$$= 4.77 \times 10^{-8} \ N/m^2$$

6. A plane em wave in vacuum has amplitude $E_0 = 50\times 10^{-6}$ V/m. Calculate
i) The avarage energy density.
ii) The peak energy density
iii) The avarage value of pointing vector
iv) The peak value of pointing vector.

Soln: i) The avarage energy density

$$\overline{U_{em}} = \frac{1}{2}\epsilon_0 |E_0|^2$$

$$= \frac{1}{2} \cancel{\times 8.85 \times 10^{-12}}$$

$$= \frac{1}{2} \times 8.85 \times 10^{-12} \times (50\times 10^{-6})^2$$

$$\overline{U_{em}} = 11062.5 \times 10^{-24} \ J/m^3$$

$$= 1.1 \times 10^{-20} \ J/m^3.$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

ii) The total energy density

$$U_{em}\Big|_{max.} = \frac{1}{2}\left(\epsilon_0|E_0|^2 + \frac{|B_0|^2}{\mu_0}\right)$$

$$= \frac{1}{2}\left[\epsilon_0|E_0|^2 + \frac{|E_0|^2}{\mu_0 c^2}\right]$$

$$= \frac{1}{2}\left[2\epsilon_0|E_0|^2\right] = \epsilon_0|E_0|^2.$$

$$= 2 \times 1.1 \times 10^{-20}\ J/m^3$$

$$= 2.2 \times 10^{-20}\ J/m^3$$

iii) The average value of pointing vector $\bar{S} = \frac{1}{2}\epsilon_0|E_0|^2 c$

$$= 1.1 \times 10^{-20} \times 3 \times 10^{8}\ J/m^2$$

$$= 3.318 \times 10^{-20}\ J/m^2$$

iv). The peak value of pointing vector

$$S\Big|_{max.} = \frac{|E_0|\,|B_0|}{\mu_0}$$

$$= \frac{|E_0|^2}{\mu_0 c}$$

$$= \epsilon_0 c\,|E_0|^2$$

$$= 2 \times \frac{1}{2}\epsilon_0 c|E_0|^2$$

$$= 2 \times \bar{S}$$

$$= 2 \times 3.31 \times 10^{-20}\ J/m^2$$

$$= 6.6 \times 10^{-20}\ J/m^2$$