Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

PROGRAMME: B.Sc. PHYSICS HONOURS

SEMESTER - VI

Course: Electromagnetic Theory (CC-XIII)

Lecture Note: 6

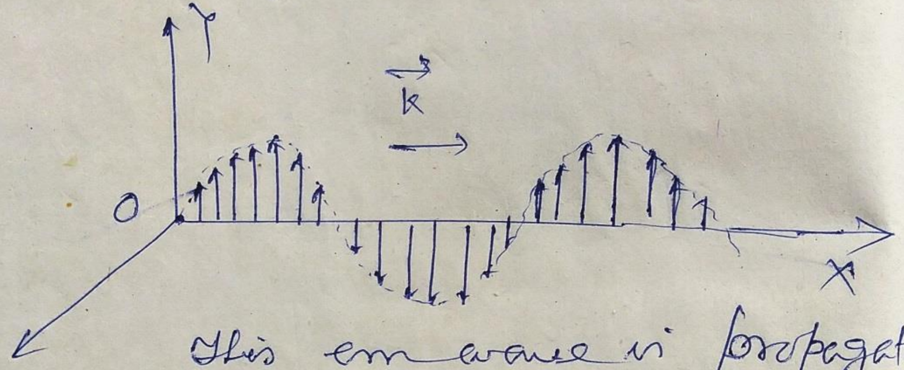
Plane Electromagnetic Wave in dielectrics-2

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

Polarization of em wave:

~~If an em wave is confined~~

Electromagnetic wave is a transverse wave, because it's displacement is always perpendicular to it's direction of propagation. If the em wave confines its vibration on a fixed plane, ~~then~~ i.e ~~the~~ wave possess a fixed plane of vibration, then the em wave is called polarized wave.



This em wave is propagating along X ~~dir~~ direction. The plane of vibration is X-Y plane but the plane of polarization which is perpendicular to plane of vibration is X-Z plane. The plane of polarization is the plane about which the wave is ~~a~~ symmetry. The mathematical equation of a plane polarized wave

$$\vec{E} = \hat{j} E_0 \cos(kz - \omega t + \phi).$$

where E_0 = amplitude, ~~k~~ ω and ϕ

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

... angular frequency and phase const.

State of polarization :-

There are four states of polarization such as
i) linearly polarized wave.
ii) Circular polarized ",
iii) ~~Elastically~~ Elleptically polarized wave
iv) Unpolarized wave.

~~linear~~ Linear Polarized wave :-

For a linearly polarized wave the ~~B~~ vibration of ~~electric~~ vector is confined in a definite plane while the wave progressing through the medium. The general mathematical form of a plane ~~per~~ progressive wave travelling along +Z axis with angular frequency ω is written as

$$\vec{E} = \left(\hat{i}\, E_{10} + \hat{j}\, E_{20}\right) \cos\,(kz - \omega t).$$

where $E_{10} = E_0 \cos\theta$, $E_{20} = E_0 \sin\theta$.

the components of amplitude along X and Y direction. The math. eqn of a linearly polarized wave travelling through Z axis with frequency ω, amplitude E_0 with it's vibration lying on the x-z plane is given

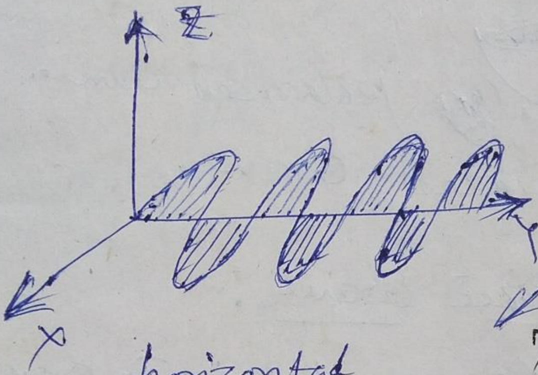
Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

by $\vec{E} = \hat{i} E_0 \cos(kz - \omega t)$.

There are two ~~types of~~ independent states of linearly polarized wave.

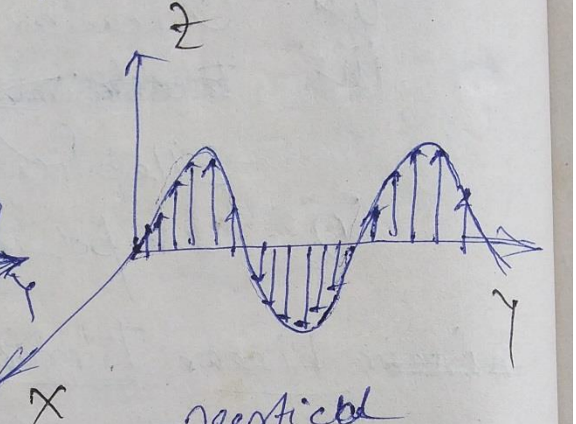
i) horizontal polarization.

ii) vertical polarization.



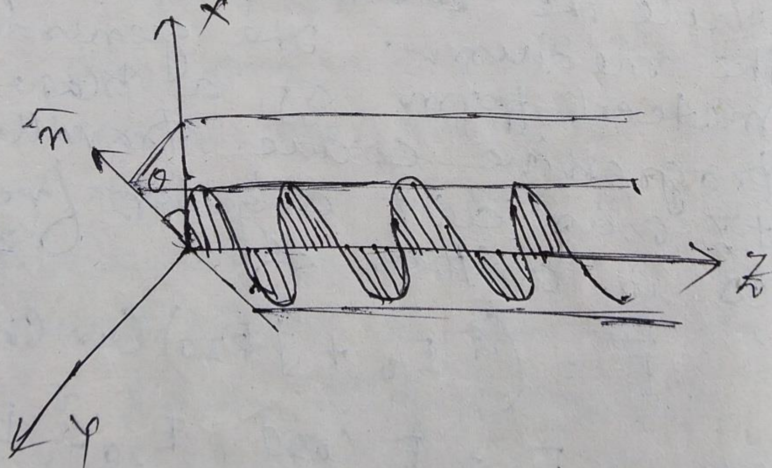
horizontal polarization

$\vec{E} = \hat{i} E_0 \cos(kz - \omega t)$



vertical polarization.

$\vec{E} = \hat{k} E_0 \cos(kz - \omega t)$



It represents a plane polarised wave progressing along z direction through the medium. It's plane of vibration makes an angle θ with XZ plane. The direction of polarization

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

or polarization vector

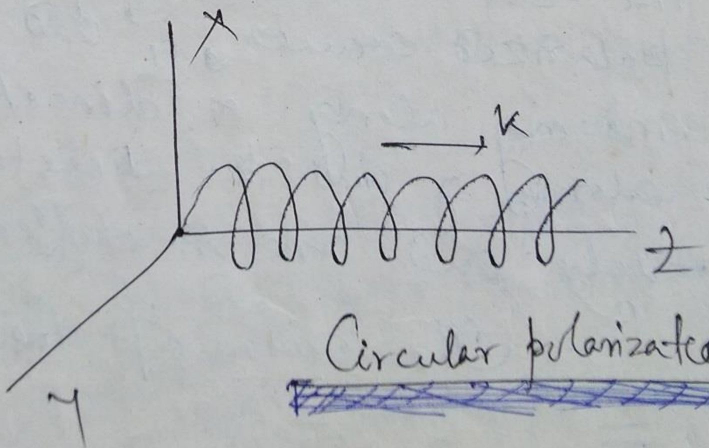
$$\hat{n} = \hat{i}\cos\theta + \hat{j}\sin\theta.$$ ~~defined~~

defines the plane of vibration.

Circular Polarization:-

When two independent linearly polarized ~~electrical~~ em wave such as light of same frequency or wavelength and same direction of propagation and same amplitude and different in phase $\pi/2$ are superimposed, the resultant wave has the constant magnitude rotating in circular path (anticlockwise) while moving. The mathematical equation of circular polarization

$$\vec{E} = \hat{i}E_0\sin(kz-\omega t) + \hat{j}E_0\cos(kz-\omega t).$$



Circular polarization

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

Elliptically polarised wave :-

We know that a linearly polarised wave may be considered a superposition of two waves of same amplitude and phase, one vertically and other horizontally polarized.

$$\therefore E(x,t) = E_0 \cos\phi \, e^{j(kx-\omega t)} \hat{j} + E_0 \sin\phi \times e^{+j(kx-\omega t)} \hat{k}.$$

$$= E_0 e^{\hat{j}(kx-\omega t+\phi)} \hat{n}.$$

The elliptically or circularly polarized wave can be constructed by the superposition of two plane polarized waves, which are not in same phase. Let us consider the the electric vectors of two linearly polarized waves, $\vec{E_1}$ and $\vec{E_2}$, both moving along x direction, polarized along y and z direction respectively and are in different phases.

The sum of these two vectors

$$\bar{E} = \bar{E_1} + \bar{E_2}$$

$$= \hat{j} E_{10} \cos(\omega t - kx - \phi_1) + \hat{k} E_{20} \cos(\omega t - kx - \phi_2). \quad \text{--- (1)}$$

If $\phi_1 \neq \phi_2$, the resultant wave $\vec{E}$ are said to be elliptically

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

polarised.

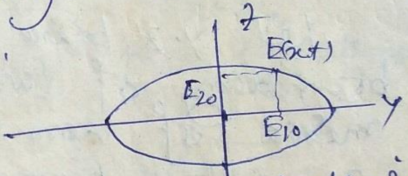
A special case of polarization is when $\phi_1 = \phi$, $\phi_2 = \phi \pm \pi/2$. then we have

$$\bar{E} = \hat{j} E_{10} \cos(\omega t - kx - \phi) + \hat{k} E_{20} \cos(\omega t - kx - \phi \mp \pi/2)$$
$$= \hat{j} E_{10} \cos(\omega t - kx - \phi) \pm \hat{k} \sin(\omega t - kx - \phi)$$
$$= \hat{j} E_y + \hat{k} E_z$$

Hence we have $E_y = E_{10} \cos(\omega t - kx - \phi)$

$E_z = E_{20} \sin(\omega t - kx - \phi)$.

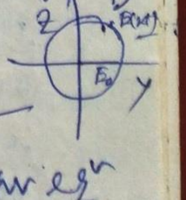
$\therefore \left(\frac{E_y}{E_{10}}\right)^2 + \left(\frac{E_z}{E_{20}}\right)^2 = 1$ which is an equation of an ellipse with major axis E_{10} and minor axis E_{20}.



The electric vector moves clockwise or anticlockwise depending upon the sign (+) or (−) in $\phi_2 = \phi \pm \pi/2$

Circular polarization is special case of elliptical polarization where $E_{10} = E_{20} = E_0$. Thus for circularly polarized wave

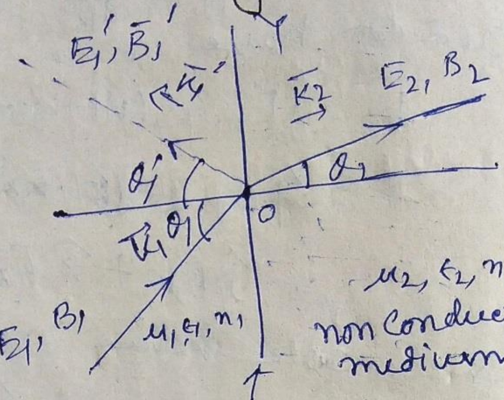
$$\bar{E} = E_0 \left[\hat{j} \cos(\omega t - kx - \phi) \pm \hat{k} \sin(\omega t - kx - \phi)\right]$$
$$= E_y \hat{j} + E_z \hat{k}.$$



$\therefore \left(\frac{E_y}{E_0}\right)^2 + \left(\frac{E_z}{E_0}\right)^2 = 1$ or $E_y^2 + E_z^2 = E_0^2$ which is an eq^n of a circle.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

Reflection and refraction of em wave by a non-conducting boundary:



Let Y-Z plane is the plane of boundary of two transparent linear media of zero conductivity (dielectric) absolute permeability and permitivity μ_1 & ϵ_1, refractive index n_1 and for another the same are μ_2, ϵ_2, n_2, as shown in fig.

Let E_{10}, E_{10}' and E_{20} are the amplitude of incident waves ($\bar{E}_1$), reflected wave ($\bar{E}_1'$) and refracted wave ($\bar{E}_2$) with propagation vector $\vec{k_1}$, $\vec{k_1'}$ and $\vec{k_2}$. The electric field of the incident wave with polarization vector $\hat{\epsilon}_1$ in the plane of incidence (~~X-Y~~) is represented by

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

$$\vec{E}_1 = \hat{\varepsilon}_1 E_{10}\, e^{j(\vec{k}_1\cdot\vec{r} - \omega t)} \quad \text{--- (1)}$$

The corresponding magnetic field of incident em wave

$$\vec{B}_1 = \frac{\vec{k}_1 \times \vec{E}_1}{\omega} \quad \text{--- (2)}$$

Similarly the reflected and refracted electric vector and magnetic vector are of similar types. i.e $\vec{E}_1'$, $\vec{B}_1'$, and $\vec{E}_2$, $\vec{B}_2$.

Very near to the surface of separation, they are satisfying condition known as boundary condition.

a) $\epsilon_1 (\vec{E}_1 + \vec{E}_1')_x = \epsilon_2 E_2)_x$. (normal component of $\vec{D}$).

b) $(\vec{B}_1 + \vec{B}_1')_x = B_2)_x$. (normal comp. of B).

c) $(E_1 + E_1')_{y,z} = E_2)_{y,z}$. (tangential comp. of E).

d) $\left.\frac{\vec{B}_1 + \vec{B}_1'}{\mu_1}\right|_{y,z} = \left.\frac{B_2}{\mu_2}\right|_{y,z}$ (tangential comp. of $\vec{H}$).

In the 1st medium there are two waves, reflected + incident, but in 2nd medium there is only one wave, refracted wave.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

These four boundary condition are true at all points of boundary at at all time t. The only way to ~~sa~~ satisfy this is that

$$\bar{K}_1 \cdot \bar{r} = \bar{k}_1' \cdot \bar{r} = \bar{k}_2 \cdot \bar{r} \text{ at all time.} \quad (3)$$

Eq^n (3) gives the propagation vector must be co-plannar. This proves the 1st law of reflection.

If $\vec{r}$ lies in y-z plane,

$$k_1 \sin\theta_1 = k_1' \sin\theta_1' = k_2 \sin\theta_2$$

as $k_1 = k_1'$, so

$$\boxed{\theta_1 = \theta_1'}$$

i.e. (the angle of incidence is equal to the angle of reflection) which 2nd law of reflection.

Again
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{k_2}{k_1} = \frac{\omega/v_2}{\omega/v_1} = \frac{v_1}{v_2} = \frac{c/v_2}{c/v_1} = \frac{n_2}{n_1} = n$$

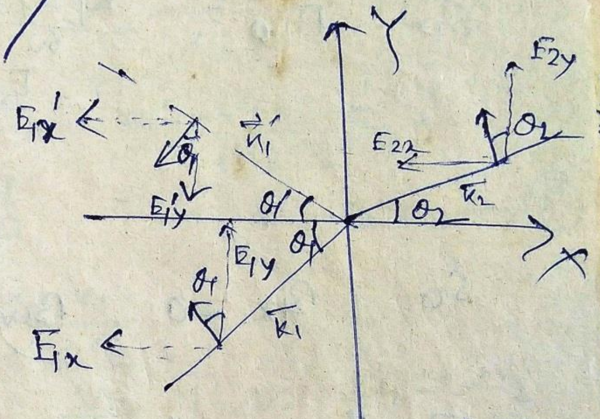
$$\therefore \boxed{\frac{\sin\theta_1}{\sin\theta_2} = n} \quad (4)$$

where n is the ~~relative~~ refractive index of 2nd w.r.to the 1st medium.
This is Snell's law of refraction.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

So Snell's ~~law~~ law of refraction of light is true for em wave. In fact all the ~~la~~ laws of reflection and refraction are also true for ordinary light.

Case-I:→ Let the incident wave is polarized in the plane of incidence (i.e. x-y plane).



Resolving the amplitude in the three coordinate axis

$$\left.\begin{aligned} E_{1x} &= -E_{10}\sin\theta_1 \\ E_{1y} &= E_{10}\cos\theta_1 \\ E_{1z} &= 0. \end{aligned}\right\} \quad (5.1)$$

Similarly
$$\left.\begin{aligned} E'_{1x} &= -E'_{10}\sin\theta_1 \\ E'_{1y} &= -E'_{10}\cos\theta_1 \\ E'_{1z} &= 0 \end{aligned}\right\} \quad (5.2)$$

and
$$\left.\begin{aligned} E_{2x} &= -E_{20}\sin\theta_2 \\ E_{2y} &= E_{20}\cos\theta_2 \\ E_{2z} &= 0 \end{aligned}\right\} \quad (5.3)$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

As $\vec{B}_{1} = \dfrac{\vec{K}_1 \times \vec{E}_1}{\omega}$

$$\vec{B}_{1} = \frac{k_1 E_{10}}{\omega} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \end{vmatrix}$$

as $\vec{K}_1 = K_{1x}\hat{i} + K_{1y}\hat{j} + K_{1z}\hat{k}$

$$= K_1\cos\theta_1\,\hat{i} + K_1\sin\theta_1\,\hat{j} + 0.$$

So $\vec{B}_{1} = \dfrac{K_1 E_{10}}{\omega}\,\hat{k}\,(\cos^2\theta_1 + \sin^2\theta_1)$

$$= \frac{E_{10} K_1}{\omega}\,\hat{k}$$

$$= \frac{E_{10}}{V_1}\,\hat{k} \quad \left[V_1 = \omega/K_1 \right]$$

So $B_{1x} = 0, \quad B_{1y} = 0, \quad B_{1z} = \dfrac{E_{10}}{V_1}$ ——— (6.1)

Similarly $B_{1x}' = 0, \; B_{1y}' = 0, \; B_{1z}' = \dfrac{E_{10}'}{V_1'}$ ——— (6.2)

$[V_1' = V_1]$

and $B_{2x} = 0, \; B_{2y} = 0, \; B_{2z} = \dfrac{E_{20}}{V_2}$ ——— (6.3)

Hence from boundary Condition

$$E_{10} + E_{10}'\big|_y = E_{20}\big|_y$$

$$\Rightarrow E_{10}\cos\theta_1 - E_{10}'\cos\theta_1 = E_{20}\cos\theta_2$$

$$\Rightarrow (E_{10} - E_{10}')\cos\theta_1 = E_{20}\cos\theta_2$$

$$\Rightarrow E_{10} - E_{10}' = E_{20}\cdot\frac{\cos\theta_2}{\cos\theta_1}$$

$$= \alpha E_{20}. \quad \text{——— (7)}$$

where $\alpha = \dfrac{\cos\theta_2}{\cos\theta_1}$ is a real no.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

and it is $\simeq 1$. when $\theta_1 \to 0$. i.e for ~~nom~~ normal incidence —

The other boundary condition

$$\left.\frac{B_{10} + B_{10}'}{\mu_1}\right|_z = \left.\frac{B_{20}}{\mu_2}\right|_z$$

$$\Rightarrow \frac{E_{10} + E_{10}'}{v_1 \mu_1} = \frac{E_{20}}{\mu_2 v_2}$$

$$\Rightarrow E_{10} + E_{10}' = \frac{\mu_1 v_1}{\mu_2 v_2} E_{20}$$

$$\Rightarrow E_{10} + E_{10}' = \beta E_{20} \quad \text{--- (8)}$$

where $\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$

$\simeq \frac{v_1}{v_2}$ [as $\mu_1 \simeq \mu_2$ other than fero-magnetic material]

$= \frac{n_2}{n_1}$

$\Rightarrow \beta = n$, relative re. index of 2nd w.r.to 1st.

Solving eq^n (7) and (8) we get

$$\left.\begin{aligned} E_{10} &= \left(\frac{\alpha+\beta}{2}\right) E_{20} \\ E_{10}' &= \left(\frac{\beta-\alpha}{2}\right) E_{20} = -\left(\frac{\alpha-\beta}{2}\right) E_{20} \end{aligned}\right\} \text{-(9)}$$

$$\therefore E_{10}' = -\left(\frac{\alpha-\beta}{\alpha+\beta}\right) E_{10} \quad \text{--- (10)}$$

So eq^n (9) and (10) are the Fresnel's formular for reflection and refraction of em wave from a dielectric boundary.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

So $E_{10}' = -ve$ for $\alpha > \beta$.
$= +ve$ for $\alpha < \beta$.

So there appear a phase change of 180° of the ~~refl~~ reflected ~~wave~~ wave provided $\alpha < \beta$ or $n > 1$. i.e reflected by a densor medium.

So from eq^n 8

$$E_{20} = \frac{1}{\beta}\left[E_{10} + E_{10}'\right]$$
$$= \frac{1}{\beta}\left[1 - \frac{\alpha-\beta}{\alpha+\beta}\right] E_{10}$$
$$E_{20} = \left(\frac{2}{\alpha+\beta}\right) E_{10}$$

This is also the Fresnel's formula for refraction of em wave.

In terms of angle the Fresnel's equation gives,

the amplitude of transmitted wave from 8

$$E_{20} = \left(\frac{2}{\alpha+\beta}\right) E_{10}$$
$$= \frac{2}{\frac{\cos\theta_2}{\cos\theta_1} + \frac{\sin\theta_1}{\sin\theta_2}} E_{10}$$
$$= \frac{2\cos\theta_1 \sin\theta_2}{\sin\theta_1\cos\theta_1 + \cos\theta_2\sin\theta_2} E_{10}$$
$$E_{20} = \frac{2\cos\theta_1 \sin\theta_2}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)} E_{10}$$

(11)

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

The amplitude of reflected wave.

$$E_{10}' = -\frac{(\alpha-\beta)}{(\alpha+\beta)} E_{10}$$

$$= -\frac{\left(\frac{\cos\theta_2}{\cos\theta_1} - \frac{\sin\theta_1}{\sin\theta_2}\right)}{\left(\frac{\cos\theta_2}{\cos\theta_1} + \frac{\sin\theta_1}{\sin\theta_2}\right)} E_{10}$$

$$= -\left(\frac{\cos\theta_2 \sin\theta_2 - \cos\theta_1 \sin\theta_1}{\cos\theta_2 \sin\theta_2 + \cos\theta_1 \sin\theta_1}\right) E_{10}$$

$$= \frac{\cos(\theta_1+\theta_2)\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)} E_{10}$$

$$\therefore E_{10}' = \frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)} E_{10} \quad \text{—— (12)}$$

So the eq^n (11) and (12) also represents the ~~Fresl~~ Fresnel's formulae but in different form.

When $\theta_1 \simeq 90^\circ$ (grazing incidence) it is seen $E_{20} = 0$. i.e. no transmission but ~~to reflect~~ total reflection. ~~from~~

On the other hand if $(\theta_1+\theta_2) = \pi/2$, then $E_{10}' = 0$.

This particular angle of incidence for which the reflected light is completely extinguished, is known as Brewster's angle, denoted by θ_B. ~~So~~.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

So at Brewster's angle $\theta_1 = \theta_B$
The refractive index of the 2nd w.r.to 1st is $n = \dfrac{\sin\theta_1}{\sin\theta_2}$

$$= \frac{\sin\theta_B}{\sin(\pi/2 - \theta_B)} \quad \because [\theta_2 = \pi/2 - \theta_B$$

$$= \frac{\sin\theta_B}{\cos\theta_B}$$

$$\boxed{n = \tan\theta_B}$$

or $\theta_B = \tan^{-1}(n)$

or $\theta_B = \tan^{-1}(n_2/n_1)$ —— (13)

Now for air to glass

$n_2 = 1.5$

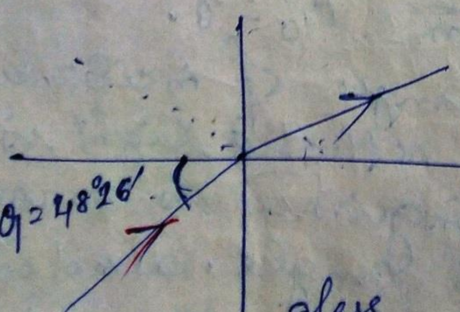
$n_1 = 1.00$

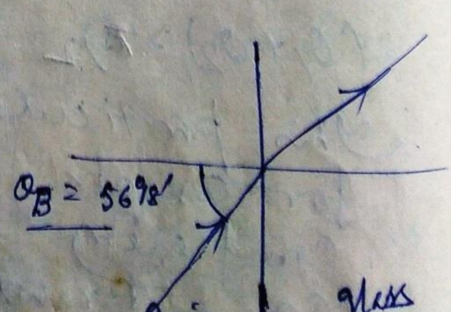
$\therefore \theta_B = \tan^{-1}(1.5) = 56^\circ 18' 35.8''$

and for water to glass.

$$\theta_B = \tan^{-1}\left(\frac{1.5}{1.33}\right)$$

$$\theta_B = 48^\circ 26' 15.6''$$





no reflection but total transmission.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

So at Brewster angle (θ_B) the angle between the reflected beam and transmitted beam is $\pi/2$.

The em power per unit area striking the boundary surface is given by

$$\vec{S}\cdot\hat{n} = S\cos\theta,$$

where $\vec{S}$ is the pointing vector, $\hat{n}$ is unit normal to the surface.

So the incident intensity of em wave is $I_1 = \vec{S}\cdot\hat{n}$.

$$I_1 = \frac{1}{2}\epsilon_1 v_1 |E_{10}|^2 \cos\theta_1 \text{ (medium)}$$

Similarly the reflected intensity

$$I_1' = \frac{1}{2}\epsilon_1 v_1 |E_{10}'|^2 \cos\theta_1$$

The transmitted intensity

$$I_2 = \frac{1}{2}\epsilon_2 v_2 |E_{20}|^2 \cos\theta_2$$

The reflection coefficient of the surface (i.e. part of light intensity reflected) is

$$R = \frac{I_1'}{I_1}$$

$$= \left|\frac{E_{10}'}{E_{10}}\right|^2 = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2 \quad (14)$$

for normal incidence $\alpha = 1$, $\beta = n$, rel. re. index of 2nd w.r.to 1st.

$$R = \left(\frac{1-n}{1+n}\right)^2 = \left(\frac{n-1}{n+1}\right)^2 \quad (15)$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

The transmission Coefficient of the surface

$$T = \frac{I_2}{I_1}$$

$$= \frac{\epsilon_2 v_2}{\epsilon_1 v_1} \cdot \left|\frac{E_{20}}{E_{10}}\right|^2 \frac{\cos\theta_2}{\cos\theta_1}$$

$$= \frac{\epsilon_2}{\epsilon_1} \cdot \frac{n_1}{n_2} \left|\frac{E_{20}}{E_{10}}\right|^2 \cdot \alpha$$

$$\boxed{\begin{array}{l} \frac{\epsilon_2}{\epsilon_1} = \left(\frac{n_2}{n_1}\right)^2 \\ n = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \end{array}}$$

$$= \cancel{\alpha\beta \left(\frac{2}{\alpha+\beta}\right)^2}$$

$$= \left(\frac{n_2}{n_1}\right)^2 \cdot \left(\frac{n_1}{n_2}\right) \left(\frac{2}{\alpha+\beta}\right)^2 \alpha.$$

$$= \frac{n_2}{n_1} \cdot \alpha \left(2/\alpha+\beta\right)^2$$

$$T = \alpha\beta \left(\frac{2}{\alpha+\beta}\right)^2 \quad \text{--- (16)}$$

for normal incidence

$$T = n \left(\frac{2}{1+n}\right)^2.$$

Now $R + T$

$$= \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2 + \alpha\beta \left(\frac{2}{\alpha+\beta}\right)^2$$

$$= \frac{1}{(\alpha+\beta)^2} \left[(\alpha+\beta)^2\right] = 1$$

So ~~the~~ sum of Reflection & transmission Coefficient is 1.

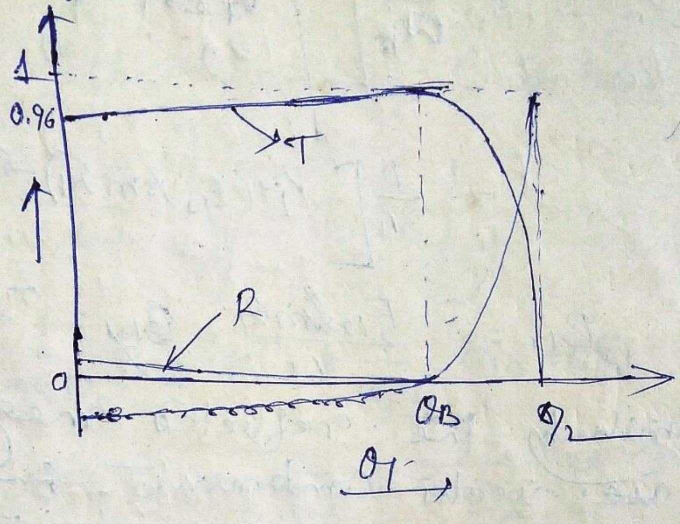
The reflection efficient $R = 0$, at $\alpha = \beta$.

i.e at $\theta_1 = \theta_B$

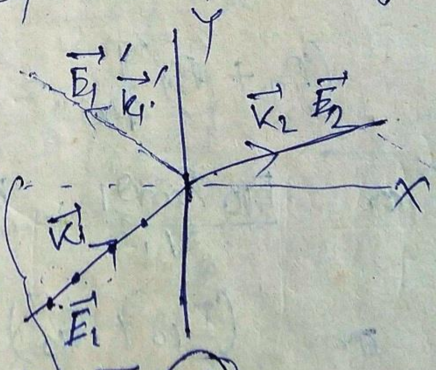
Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

and $T = 1$ at $\theta_1 = \theta_B$.

A plot of T or R for a given boundary for different values of angle of incidence is shown in fig.



Case II :- If the incident wave is polarised in the plane perpendicular to the plane of incidence i.e. along Z axis, then the component of $\vec{E}$ field vector (except the phase factor) are



$E_{1x} = 0$, $E_{1y} = 0$, $E_{1z} = E_{10}$

$E_{1x}' = 0$ $E_{1y}' = 0$, $E_{1z}' = E_{10}'$

and

$E_{2x} = 0$, $E_{2y} = 0$ $E_{2z} = E_{20}$ — (17)

Now the component of incident magnetic field along x, y, z direction

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

$$B_1 = \frac{\vec{k}_1 \times \vec{E}_1}{\omega}$$

$$= \frac{1}{v_1}\left[\hat{k}_1 \times \vec{E}_1\right]$$

$$= \frac{1}{\omega}\left[\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ k_1\cos\theta_1 & k_1\sin\theta_1 & 0 \\ 0 & 0 & E_{10} \end{vmatrix}\right]$$

$$= \frac{1}{v_1}\left[\hat{i}(E_{10}\sin\theta_1) - \hat{j}\,E_{10}\cos\theta_1\right]$$

$\therefore$ $B_{1x} = \dfrac{E_{10}\sin\theta_1}{v_1}$, $B_{1y} = \dfrac{-E_{10}\cos\theta_1}{v_1}$, $B_{1z} = 0$

Similarly, the reflected magnetic field ~~are~~ vector components are

$B'_{1x} = \dfrac{E'_{10}\sin\theta_1}{v_1}$, $B'_{1y} = +\dfrac{E'_{10}\cos\theta_1}{v_1}$, $B'_{1z} = 0$

Same for transmitted mag. field.

$B_{2x} = \dfrac{E_{20}\sin\theta_2}{v_2}$, $B_{2y} = \dfrac{-E_{20}\cos\theta_2}{v_2}$, $B_{2z} = 0$ — (8)

From the 2nd boundary condition

$$(B_1 + B'_1)\big|_x = B_2\big|_x \quad \text{(normal component of } B\text{)}$$

or $$\frac{E_{10}\sin\theta_1}{v_1} + \frac{E'_{10}\sin\theta_1}{v_1} = \frac{E_{20}\sin\theta_2}{v_2}$$

or $$(E_{10} + E'_{10}) = \frac{v_1}{v_2}\frac{\sin\theta_2}{\sin\theta_1} E_{20}.$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

From the 3rd boundary condition.

$$(E_1 + E_1')\big|_z = E_2\big|_z$$

$$\Rightarrow \quad E_{10} + E_{10}' = E_{20} \quad \text{——— (19)}$$

From the 4th boundary condition

$$\frac{B_1 + B_1'}{\mu_1}\bigg|_y = \frac{B_2}{\mu_2}\bigg|_y$$

$$\Rightarrow -\frac{1}{\mu_1 v_1} E_{10} \cos\theta_1 + \frac{1}{\mu_1 v_1} E_{10}' \cos\theta_1 = -\frac{1}{\mu_2 v_2} E_{20} \cos\theta_2$$

$$\Rightarrow \quad E_{10} - E_{10}' = \frac{\mu_1 v_1}{\mu_2 v_2} \frac{\cos\theta_2}{\cos\theta_1} E_{20}$$

$$\Rightarrow \quad E_{10} - E_{10}' = \alpha\beta\, E_{20} \quad \text{—— (20)}$$

Solving eqn (19) and (20) we get

$$E_{10} = \left(\frac{1+\alpha\beta}{2}\right) E_{20}$$

or

$$E_{20} = \left(\frac{2}{1+\alpha\beta}\right) E_{10} \quad \text{—— (21)}$$

∴ from (19) we get

$$E_{10}' = E_{20} - E_{10}$$

$$= \left(\frac{2}{1+\alpha\beta} - 1\right) E_{10}$$

$$E_{10}' = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right) E_{10}$$

—— (22)

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

Eqn. (21) and (22) are the Fresnel's formulae for reflection and refraction.

In terms of angle the ~~present~~ Fresnel's formulae

$$E_{10}' = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right) E_{10}$$

$$= \frac{1 - \frac{\cos\theta_2}{\cos\theta_1}\cdot\frac{\sin\theta_1}{\sin\theta_2}}{1 + \frac{\cos\theta_2}{\cos\theta_1}\cdot\frac{\sin\theta_1}{\sin\theta_2}} E_{10}$$

$$= \frac{\cos\theta_1 \sin\theta_2 - \cos\theta_2 \sin\theta_1}{\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1} E_{10}$$

$$E_{10}' = -\frac{\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)} E_{10} \qquad (23)$$

In denser medium $\theta_1 > \theta_2$, so there appear a phase change of $180°$ in reflected light.

Now $$E_{20} = \left(\frac{2}{1+\alpha\beta}\right) E_{10}$$

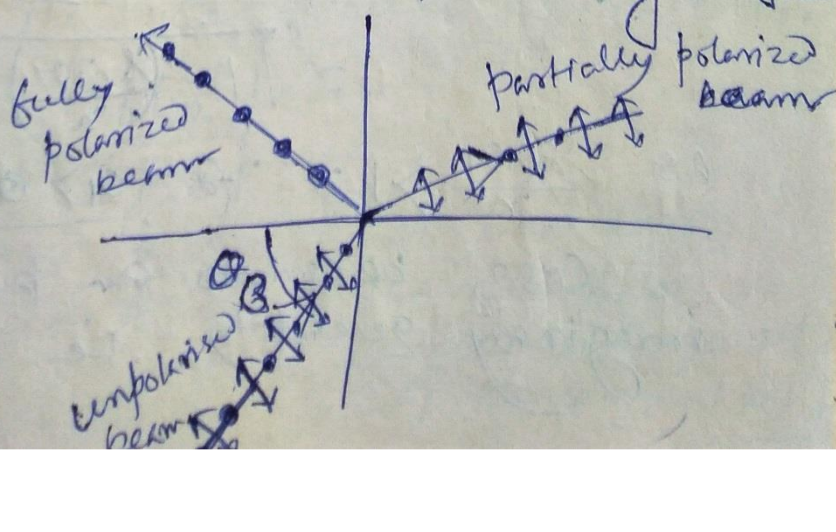
$$= \frac{2\cos\theta_1 \sin\theta_2}{\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2} E_{10}$$

$$E_{20} = \frac{2\sin\theta_2 \cos\theta_1}{\sin(\theta_1+\theta_2)} E_{10} \qquad (24)$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

It is seen from eq (23) $E'_{10} \neq 0$, never. So if the light is polarized perpendicular to the plane of incidence, there is no Brewster angle (θ_B). If $\theta_1 = \theta_2$, then $n_1 = n_2$ i.e the two media are optically indistinguishable.

* If a natural light (unpolarized) is incident on the boundary at Brewster angle, then the amplitude of electric wave for the reflected light has no component in the plane of incidence, but the component of electric vector perpendicular to the plane of incidence is reflected. This means the reflected light wave for an unpolarized incident beam is polarized if the angle of incidence is equal to the Brewster angle θ_B. This is reason why θ_B is also called the polarizing angle.

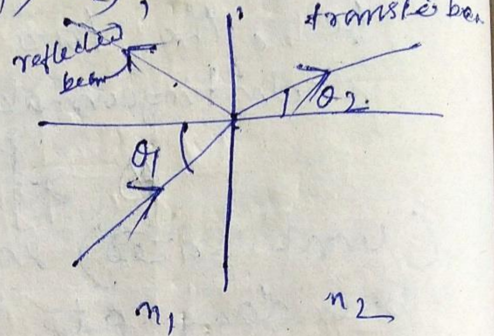


Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

Total internal Reflection :-

When light or em. wave is a incident from a optically denser medium to a rarer medium i.e ($n_1 > n_2$),

The refractive index of the 2nd medium with respect to the 1st



$$n = \frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} < 1 \qquad \underline{n_1 > n_2}$$

i.e $\theta_2 > \theta_1$

or $\sin\theta_2 = \frac{\sin\theta_1}{n}$

At the critical angle, θ_c,

$$\theta_1 = \theta_c, \quad \theta_2 = \pi/2 .$$

$$\therefore \quad \boxed{\sin\theta_c = n}$$

Now for the angle of incidence $\theta_1 > \theta_c$.

Then $\cos\theta_2 = \sqrt{1 - \sin^2\theta_2}$

$$= \sqrt{1 - \left(\frac{\sin\theta_1}{n}\right)^2}$$

As $\frac{\sin\theta_1}{n} > 1$, for $\underline{\theta_1 > \theta_c}$. then $\cos\theta_2$ becomes an pure ~~im~~ imaginary quantity. i.e we can

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

write $\cos\theta_2 = j\sqrt{\left(\frac{\sin\theta_1}{n}\right)^2 - 1}$

where, $j = \sqrt{-1}$ ——— (1)

Now we know that the amplitude of reflected light is

$$E_{10}' = \left(\frac{\alpha - \beta}{\alpha + \beta}\right) E_{10},$$

where $\alpha = \frac{\cos\theta_2}{\cos\theta_1}$ and $\beta = \frac{\sin\theta_1}{\sin\theta_2} = n$.

E_{10} = amplitude of incident light.

So $$E_{10}' = \frac{\left(\frac{\cos\theta_2}{\cos\theta_1} - \frac{\sin\theta_1}{\sin\theta_2}\right)}{\left(\frac{\cos\theta_2}{\cos\theta_1} + \frac{\sin\theta_1}{\sin\theta_2}\right)} E_{10}$$

$$= \left(\frac{\sin\theta_2\cos\theta_2 - \sin\theta_1\cos\theta_1}{\sin\theta_2\cos\theta_2 + \sin\theta_1\cos\theta_1}\right) E_{10}$$

puting the value of $\cos\theta_2$, from eqn (1) we get

$$E_{10}' = E_{10}\frac{\left(j\sin\theta_2\sqrt{\left(\frac{\sin\theta_1}{n}\right)^2 - 1} - \sin\theta_1\cos\theta_1\right)}{\left(j\sin\theta_2\sqrt{\left(\frac{\sin\theta_1}{n}\right)^2 - 1} + \sin\theta_1\cos\theta_1\right)} \quad \text{—— (2)}$$

Now the coefficient of reflection

$$R = \left|\frac{E_{10}'}{E_{10}}\right|^2$$

$R = 1$ ——— (3) [as from eqn (2) the numinator is complex conjugate of dinumination].

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

The equation ③ shows, when the angle of incidence ~~$\theta_1 > \theta_c$~~ greater than critical angle (i.e $\theta_1 > \theta_c$), the reflection coefficient is unity i.e the total incident light is reflected and there is no transmission. This pheomenon is called total internal reflection.

The two component of light wave suffer a phase change with respect to incident light on total internal reflection. The resultant light is then elliptically polarized (although the incident beam is unpolarized).

The average rate of energy flow in the 2nd medium

$\langle \vec{S}\cdot\hat{n} \rangle$ $\hat{n}$ = is the unit vector normal to surface

$$= \frac{1}{2} \mathrm{Re}\left(\frac{\vec{E}_2 \times \vec{B}_2^*}{\mu_2}\right)\cdot\hat{n}$$

$$= \frac{1}{2} \mathrm{Re}\, \frac{\vec{E}_2 \times (\vec{k}_2 \times \vec{E}_2^*)}{\mu_2 \omega}\cdot\hat{n}$$

$$\left[\text{as } \vec{B}_2 = \frac{\vec{k}_2 \times \vec{E}_2}{\omega}\right]$$

$$= \frac{1}{2} \mathrm{Re}\left\{\frac{\vec{k}_2(\vec{E}_2\cdot\vec{E}_2^*) - \vec{E}_2^*\underbrace{(\vec{E}_2\cdot\vec{k}_2)}_{0}}{\mu_2\omega}\right\}\cdot\hat{n}$$

$$= \frac{1}{2} \frac{\mathrm{Re}\,|E_{20}|^2}{\mu_2\omega}(\vec{k}_2\cdot\hat{n}) \quad [\text{as } \vec{E}_2\cdot\vec{k}_2 = 0]$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

$$= \frac{1}{2} \frac{Re\,|E_{20}|^2}{\mu_2 \omega} K_2 \cos\theta_2$$

As $\cos\theta_2$ is pure imaginary

$\cos\theta_2 = j\sqrt{\left(\frac{\sin\theta_1}{n}\right)^2 - 1}$ at $\theta_1 > \theta_c$

then the real part of $\langle \vec{S}\cdot\hat{n}\rangle$ must be zero.

i.e $\langle \vec{S}\cdot\hat{n}\rangle = 0$ [no real part there]

This indicates there is no energy flow across the surface, but there exist the electric field which is rapidly ~~decreasin~~ decreasing.

The electric field in the 2nd medium $\vec{E}_2 = \hat{\varepsilon}_2 E_{20}\, e^{j(\vec{k}_2\cdot\vec{r} - \omega t)}$.

Now $\vec{k}_2\cdot\vec{r} = k_2 r\cos\theta_2 = j k_2 r\sqrt{\left(\frac{\sin\theta_1}{n}\right)^2 - 1}$
$= j\phi$.

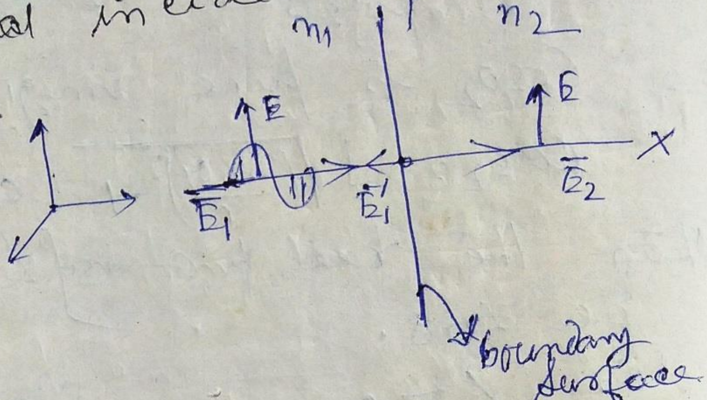
So $\vec{E}_2 = \hat{\varepsilon}_2 \cdot E_{20}\, e^{j(j\phi - \omega t)}$

$$= \frac{\hat{\varepsilon}_2 E_{20}\, e^{-j\omega t}}{e^{\phi}}$$

As ϕ (r increases ϕ increases) so the electric field vector is decreases rapidly as r increases.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

(*) Find the Fresnel's formula for normal incidence.



Consider an electromagnetic wave of frequency ω propagating along x direction, normal to the plane (YZ) interface of two linear dielectric material of propagation constant $\vec{k}_1$ and $\vec{k}_2$ ~~and~~ refractive ~~index~~ indices n_1 and n_2. The amplitude E_{10} is polarised along y-axis. The reflected waves travel in the negative x-direction while the refracted wave travels in the x-direction. We thus have for the incident wave

$$E_{1y} = E_{10}\, e^{i(k_1 x - \omega t)}$$

$$E_{1y} = E_{10}, \quad E_{1x} = 0, \quad E_{1z} = 0.$$

For reflected rays,

$$E_{1x}' = 0 \quad E_{1y}' = E_{10}', \text{ and } E_{1z}' = 0.$$

for transmitted wave

$$E_{2x} = 0, \quad E_{2y} = E_{20}, \quad E_{2z} = 0 \qquad (1)$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

The incident magnetic field.

$$B_1 = \frac{\vec{k}_1 \times \vec{E}_1}{\omega}$$

$$= \frac{1}{\omega} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ k_1 & 0 & 0 \\ 0 & E_{10} & 0 \end{vmatrix}$$

$$= \frac{k_1}{\omega} \left[\hat{k} E_{10} \right]$$

$$= \frac{E_{10}}{v_1} \hat{k}.$$

So $B_{1x} = 0$, $B_{1y} = 0$, $B_{1z} = \frac{E_{10}}{v_1}$

Similarly for reflected rays.

$B_{1x} = 0$, $B_{1y} = 0$, $B_{1z} = -\frac{E_{10}'}{v}$ (2)

Similarly for transmitted ray

$B_{2x} = 0$, $B_{2y} = 0$ $B_{2z} = \frac{E_{20}}{v}$.

From the 2nd Boundary Condition that the tangential component of $\vec{E}$ is continuous near to surface of separation $(x=0)$.

$$E_1 + E_1' \Big|_y = E_2 \Big|_y$$

$$\Rightarrow E_{10} + E_{10}' = E_{20} \longrightarrow (3)$$

again, from the 4th boundary condition, that the tangential component of magnetic intensity vector $(\vec{H})$ is continuous near to surface of separation across the boundary.

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

So we get

$$\left.\frac{B_1 + B_1'}{\mu_1}\right|_z = \left.\frac{B_2}{\mu_2}\right|_z$$

$$\Rightarrow \frac{1}{\mu_1 v_1}\left[E_{10} - E_{10}'\right] = \frac{1}{\mu_2 v_2} E_{20}$$

$$\Rightarrow E_{10} - E_{10}' = \frac{\mu_1 v_1}{\mu_2 v_2} E_{20}$$

$$\Rightarrow E_{10} - E_{10}' = \beta E_{20}. \quad \text{——} (4)$$

where $\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$

$= \frac{v_1}{v_2}$ [$\mu_1 \simeq \mu_2$ for other than feromagnetic material]

$= \frac{n_2}{n_1} = n,$

β = relative re. index of the 2nd w.r.to the 1st medium.

Solving eq^n (3) and (4)

$$E_{10} = \frac{(1+\beta)}{2} E_{20}$$

$$\Rightarrow E_{20} = \left(\frac{2}{1+\beta}\right) E_{10}$$

So $E_{10}' = \frac{(1-\beta)}{2} E_{20}$

$$\text{or } E_{10}' = \left(\frac{1-\beta}{1+\beta}\right) E_{10} \quad \text{——} (5)$$

$$\text{and } E_{20} = \left(\frac{2}{1+\beta}\right) E_{10} \quad \text{——} (6)$$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

So eq. (5) and (6) gives the amplitude of reflected and transmitted ~~light~~ wave for normal incidence. These are the Fresnel's formulae for normal incidence.

The intensity of incident light

$$I_1 = \frac{1}{2}\epsilon_1 v_1 |E_{10}|^2$$

and the intensity of reflected wave

$$I_1' = \frac{1}{2}\epsilon_1 v_1 |E_{10}'|^2.$$

and the intensity of transmitted light wave

$$I_2 = \frac{1}{2}\epsilon_2 v_2 |E_{20}|^2.$$

So the coefficient of reflection for normal incidence

$$R = \left|\frac{E_{10}'}{E_{10}}\right|^2$$

$$= \left|\frac{1-\beta}{1+\beta}\right|^2 = \left|\frac{n_1-n_2}{n_1+n_2}\right|^2 \quad (7)$$

$[\text{as } \beta = n_2/n_1]$

The transmission coefficient

$$T = \frac{I_2}{I_1} = \left(\frac{\epsilon_2 v_2}{\epsilon_1 v_1}\right)\left|\frac{E_{20}}{E_{10}}\right|^2$$

$$= \left(\frac{n_2}{n_1}\right)^2 \cdot \left(\frac{n_1}{n_2}\right) \cdot \left|\frac{2}{1+\beta}\right|^2$$

$$= \frac{n_2}{n_1}\left|\frac{2n_1}{n_1+n_2}\right|^2 \quad (8)$$

$[\text{as } \beta = n_2/n_1, \text{ as } \epsilon \propto n^2, v \propto 1/n]$

Dr. Shyamsundar Ghosh
Assistant Professor
Department of Physics, B. N. Mahavidyalaya

Now adding (7) and (8)

$$R+T = \frac{1}{(n_1+n_2)^2}\left[(n_1-n_2)^2 + 4n_1n_2\right]$$

$$= (n_1+n_2)^2/(n_1+n_2)^2$$

or $\boxed{R+T = 1}$

As the intensity expresses the average energy ~~supplied~~ crossing unit area normally, ~~per sec,~~ I_1' and I_2 are the average reflected and transmitted energy through unit area.

So the total of reflection and transmission energy

$$I_1' + I_2$$

$$= \frac{1}{2}\epsilon_1 v_1 |E_{10}'|^2 + \frac{1}{2}\epsilon_2 v_2 |E_{20}|^2$$

$$= \frac{1}{2}\epsilon_1 v_1 \left[\left(\frac{1-\beta}{1+\beta}\right)^2 + \left(\frac{2}{1+\beta}\right)^2\right] E_{10}^2$$

$$= \frac{1}{2}\epsilon_1 v_1 \left[\frac{(1+\beta)^2}{(1+\beta)^2}\right] E_{10}^2$$

$$= \frac{1}{2}\epsilon_1 v_1 E_{10}^2$$

$= I_1 =$ av. energy of the incident light.

So the energy conservation theorem