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PROGRAMME: B.Sc. PHYSICS HONOURS

SEMESTER - VI

Course: Electromagnetic Theory (CC-XIII)

Lecture Note: 7

Plane Electromagnetic Wave in Conductor

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Plane Electromagnetic wave in a Conducting medium:-

In Conducting medium where the Conductivity σ is high, the maxwell's eq^ns are in S.I. unit-

i) $\vec{\nabla}\cdot\vec{D} = \rho_f$ free charge density
ii) $\nabla\cdot B = 0$
iii) $\nabla\times E = -\partial B/\partial t$
iv) $\vec{\nabla}\times\vec{H} = \vec{J_f} + \dot{\vec{D}}$

$\Bigg\} \longrightarrow (1)$

where, $\vec{D} = \epsilon_0\vec{E}+\vec{P}$ or $B = \mu_0(\vec{M}+\vec{H})$
$= \epsilon\vec{E}$ or $B = \mu H.$

~~electric~~ displacement vector, mag ~~electric~~ field intensity vector.

J_f = is the free current density

or $J_f = \sigma\vec{E}$, σ Conductivity, for a Conductor.

Taking divergence of eq^n (1), iv)

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J_f} + \vec{\nabla}\cdot\dot{\vec{D}}$$
$$= (\vec{\nabla}\cdot\sigma\vec{E}) + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D})$$
$$0 = \sigma(\vec{\nabla}\cdot\vec{E}) + \frac{\partial\rho_f}{\partial t}$$
$$\therefore \frac{\partial}{\partial t}(\rho_f) + \sigma(\vec{\nabla}\cdot\vec{E}) = 0$$
$$\Rightarrow \frac{\partial\rho_f}{\partial t} + \frac{\sigma\rho_f}{\epsilon_0} = 0 \quad \text{--- (2)}$$

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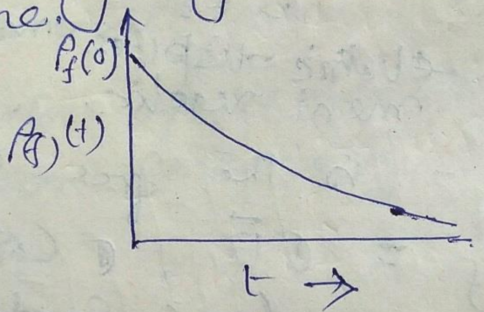
So free charge density ρ_f satisfies a ~~differen~~ 1st order partial differential equation.

or $$\frac{d\rho_f}{\rho_f} = -\frac{\sigma}{\epsilon_0}\,dt$$

or $$\rho_f(t) = \rho_f(0)\, e^{-\sigma/\epsilon_0\, t} \quad \text{——— (3)}$$

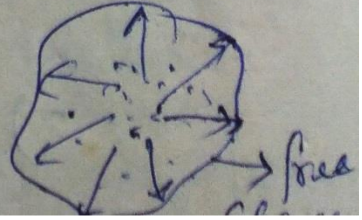
where, $\rho_f(0)$ is free charge density at the begining ie $t=0$

So the any free charge density ~~decreases~~ to the Conductor decreases very very rapidly with time.



For good Conductor σ is very high ($\sim 10^7$) and $\epsilon = 10^{-12}$, as a result $\frac{\sigma}{\epsilon_0} = 10^{19}$. So the free charge density very quickly goes over the surface of the Conductor from the volume of the Conductor.

So inside the Conductor $\rho_f = 0$,



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So Maxwell eqns in Conductor

i) $\vec{\nabla}\cdot\vec{E} = 0$

ii) $\vec{\nabla}\cdot\vec{B} = 0$

iii) $\nabla\times\vec{E} = -\partial\vec{B}/\partial t$

iv) $\vec{\nabla}\times\vec{B} = \mu\sigma\vec{E} + \mu\epsilon\frac{\partial\vec{E}}{\partial t}$

④

Taking Curl of eqn 4) iii).

$$\nabla\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$= -\frac{\partial}{\partial t}\left[\mu\sigma\vec{E} + \mu\epsilon\frac{\partial\vec{E}}{\partial t}\right]$$

$$\nabla(\vec{\nabla}\cdot\vec{E}) - \nabla^2 E = -\mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

or $$\nabla^2 E - \mu\epsilon\frac{\partial^2 E}{\partial t^2} - \mu\sigma\frac{\partial\vec{E}}{\partial t} = 0 \quad \text{—⑤}$$

[as $\vec{\nabla}\cdot\vec{E} = 0$]

Similarly ~~we also~~ get

$$\nabla^2 B - \mu\sigma\frac{\partial B}{\partial t} - \mu\epsilon\frac{\partial^2 B}{\partial t^2} = 0 \quad \text{—⑥}$$

~~The~~ eqn ⑤ and ⑥ represents a damped wave equation. So $\vec{E}$ and $\vec{B}$ wave in a Conducting ~~medium~~ medium are damped progressive wave because of presence of 1st order derivative term.

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Now the $\vec{E}$ and $\vec{B}$ field are actually waves i.e they varry harmonically with time & space. So the E wave eqn

$$\vec{E} = E_0 e^{j(\vec{k}\cdot\vec{r} - \omega t)}.$$

In electromagnetic wave the operator

$$\frac{\partial}{\partial t} \equiv -j\omega \qquad \frac{\partial^2}{\partial t^2} = -\omega^2$$

$$\vec{\nabla} \equiv j\vec{k} \qquad \nabla^2 = -k^2.$$

So the equation (5) gives

$$\nabla^2 E = \mu\sigma\frac{\partial E}{\partial t} + \mu\epsilon\frac{\partial^2 E}{\partial t^2}$$

$$= [\mu\sigma(-j\omega) + \mu\epsilon(-\omega^2)]\vec{E}$$

$$= -[\mu\epsilon\omega^2 + j\mu\sigma\omega]\vec{E}$$

$$\nabla^2 E = -\tilde{k}^2 \vec{E} \qquad \text{——— (7)}$$

where $\tilde{k}^2 = (\mu\epsilon\omega^2 + j\mu\sigma\omega)$

= a Complex number. ——— (8)

So in a conducting medium the propagation vector $\vec{k}$ is a complex number.

$$\tilde{k} = \cancel{\mu\epsilon\omega^2 + j\mu\sigma\omega}$$

$$= \alpha + j\beta. \qquad \cancel{(9)}$$

where: $\alpha = Re(\tilde{k})$ $\cancel{= \mu\epsilon\omega^2}$,

$\beta = Im(\tilde{k})$ $\cancel{= \mu\sigma\omega}$.

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$\therefore \quad \tilde{k}^2 = \alpha^2 - \beta^2 + j2\alpha\beta.$

$$\Rightarrow \tilde{k}^2 = \mu\epsilon\omega^2 + j\mu\sigma\omega$$

$$= \mu\epsilon\omega^2\left[1 + j\frac{\sigma}{\epsilon\omega}\right]$$

$$= \mu\omega^2\tilde{\epsilon}$$

where $\tilde{\epsilon} = \epsilon\left(1 + j\frac{\sigma}{\epsilon\omega}\right)$

$= \epsilon + j\sigma/\omega$

= a complex number.

Again $\tilde{k}^2 = k_0^2(1 + j\sigma/\epsilon\omega)$, — (9)

where, $k_0^2 = \mu\epsilon\omega^2$.

Now comparing real and imaginary parts we get

$$\alpha^2 - \beta^2 = k_0^2 = \mu\epsilon\omega^2 \quad \text{— (10.1)}$$

$$\& \quad 2\alpha\beta = \mu\sigma\omega. \quad \text{— (10.2)}$$

Solving these two equations,

$$\alpha = k_0\left[\frac{1 + \sqrt{1 + (\sigma/\epsilon\omega)^2}}{2}\right]^{1/2} \quad \text{— (11.1)}$$

$$\beta = k_0\left[\frac{-1 + \sqrt{1 + (\sigma/\epsilon\omega)^2}}{2}\right]^{1/2} \quad \text{— (11.2)}$$

The real part of $\tilde{k}$ i.e α gives the propagation of wave. & the imaginary part gives the attenuation of em wave in the conductor, & β is called

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absorbtion Coefficient.

Now $\alpha^2+\beta^2$

$$= K_0^2\left[1+(\sigma/\epsilon\omega)^2\right]^{1/2} = K_0^2\sqrt{1+(\sigma/\epsilon\omega)^2} \quad \text{—(12)}$$

[Now for good conductor

$$\sigma \gg \epsilon\omega$$

i.e. $\frac{\sigma}{\epsilon\omega} \gg 1$.

Then $\alpha \simeq \beta = K_0\sqrt{\sigma/2\epsilon\omega}$

$$= \sqrt{\mu\epsilon\omega^2}\cdot\sqrt{\sigma/2\epsilon\omega}.$$

$$= \sqrt{\frac{\mu\sigma\omega}{2}} \quad \text{—(13)}$$]

It is seen that $\beta \to 0$ as $\omega \to \alpha$.

The velocity of em wave

$$\tilde{v} = \omega/\tilde{K} \quad \text{(a complex no.)}$$

The refractive index of the conducting medium

$$\tilde{n} = c/\tilde{v}$$

$$= \frac{c}{\omega}\tilde{K}$$

$$= \frac{c}{\omega}(\alpha+j\beta).$$

$$= \frac{c\alpha}{\omega}(1+j\,\beta/\alpha).$$

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or $\hat{n} = n(1+j\chi)$. —(14)

where $\chi = \beta/\alpha$ is known as attenuation index.

Now $\hat{n}^2 = n^2[1-\chi^2+2j\chi]$

$= \left(\frac{c}{\hat{v}}\right)^2$

$= c^2\mu\hat{\epsilon}$.

$= c^2\mu\left[\epsilon + j\,\sigma/\omega\right]$

or $\hat{n}^2 = c^2\left[\mu\epsilon + j\frac{\sigma\mu}{\omega}\right]$ —(15)

Equating real & imaginary parts

$n^2(1-\chi^2) = c^2\mu\epsilon$.

& $2\chi n^2 = \frac{c^2\sigma\mu}{\omega}$.

Thus in a conducting medium the physical quantities such as propagation constant, refractive index, velocity, permittivity are represented by complex quantity. The complex number indicates there is absorbtion of em wave in the conducting medium.

The plane progressive wave solution of the damped wave equation (eq^n (5) & (6)) in a

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Conducting medium

$$\vec{E}(\vec{r},t) = \hat{\varepsilon}_1 E_0\, e^{j(\tilde{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \hat{\varepsilon}_2 B_0\, e^{j(\tilde{k}\cdot\vec{r}-\omega t)}$$

where E_0 and B_0 are complex amplitude

Hence $\vec{k} = (\alpha + j\beta)\hat{\varepsilon}_k$ (a complex vector).

$$= \sqrt{\alpha^2+\beta^2}\, e^{j\phi}\, \hat{\varepsilon}_k$$

where, $\tan\phi = \beta/\alpha$

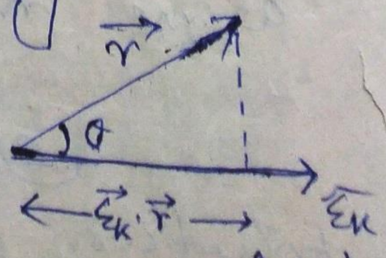
$\hat{\varepsilon}_k$ is unit propagation vector.

From (16.1)

$$\vec{E}(\vec{r},t) = \hat{\varepsilon}_1 E_0\, e^{j[(\alpha+j\beta)\hat{\varepsilon}_k\cdot\vec{r} - \omega t]}$$

or $$\vec{E}(\vec{r},t) = \hat{\varepsilon}_1 E_0\, e^{-\beta(\hat{\varepsilon}_k\cdot\vec{r})}\cdot e^{j[\alpha(\hat{\varepsilon}_k\cdot\vec{r})-\omega t]} \quad \text{--- (17)}$$

The amplitude of em wave in conducting medium is then $E_0\, e^{-\beta(\hat{\varepsilon}_k\cdot\vec{r})}$, which decreases ~~expone~~ exponentially with the penetrating distance $(\vec{\varepsilon}_k\cdot\vec{r})$ along the direction of propagation, depending on β (



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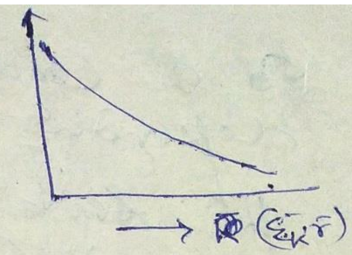

Skin depth:

The amount of penetration along $\vec{k}$ into the medium for which the initial amplitude E_0 falls to $\frac{1}{e}$ th or 36.78 % of E_0 is called skin depth. It definately depends on β and is reciprocal of β i.e. Skin depth

$$\delta = 1/\beta$$

or
$$\delta = \frac{1}{k_0 \left[\frac{-1+\sqrt{1+(\sigma/\epsilon\omega)^2}}{2}\right]^{1/2}}$$

$$= \frac{1}{\sqrt{\mu\epsilon\omega^2}}\left[\frac{2}{-1+\sqrt{1+(\sigma/\epsilon\omega)^2}}\right]^{1/2}$$

for good Conductor

$$\frac{\sigma}{\epsilon\omega} >> 1$$

$$\delta = \frac{1}{\sqrt{\mu\epsilon\omega^2}}\left(2\frac{\epsilon\omega}{\sigma}\right)^{1/2}$$

$$= \left(\frac{2}{\mu\sigma\omega}\right)^{1/2}$$

for ideal Conductor

$\sigma \to \infty$ and skin depth $\to 0$

So the skin depth of a Conductor depend on frequency ω.

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So a conductor termed as good or bad depending on frequency i.e the same substance can be a good conductor at low frequency and a poor one at high frequency.

So the conductivity of a substance depends on frequency given by

$$\boxed{\sigma = \frac{2}{\delta^2 \mu \omega}}$$

For Cu: $\sigma = 5.8 \times 10^7$ mho/m

$\omega = 2\pi \times 10^6$ Hz

$\mu = \mu_0$

$\therefore$ skin depth

$$\delta = \left(\frac{2}{\mu \sigma \omega}\right)^{1/2}$$

$$= \left(\frac{2}{4\pi \times 10^{-7} \times 5.8 \times 10^7 \times 2\pi \times 10^6}\right)^{1/2}$$

$= 0.066 \times 10^{-5}$ m $=$ ~~0.027×10^{-6} m~~

$= 0.66 \times 10^{-6}$ m

$\delta = \underline{0.66\ \mu m}$

So the value of skin depth depends on applied frequency ω and conductivity σ, but independent of dielectric const. of medium.

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The phenomenon that dynamic the field, hence the current in a conductor are confined within a very very small region ($\sim 10^{-6}$ m) of the conducting surface, is known as skin-effect.

Problem

i) ~~A~~ For Ag, $\mu = \mu_0$, $\sigma = 2\times10^{7}$ ohm^{-1} m^{-1}

Calculate skin depth.

$$\delta = \left(\frac{2}{\mu\sigma\omega}\right)^{1/2}$$

$$= \cancel{\frac{2}{4\pi\times10^{-7}\times2\times10^{7}\times10^{10}}}$$

given $f = 10^{10}$ Hz

$\therefore f = \frac{\omega}{2\pi} = 10^{10}$

$\therefore \omega = 2\pi\times10^{10}$ rad/sec

$$\delta = \left(\frac{2}{4\pi\times10^{-7}\times2\times10^{7}\times2\pi\times10^{10}}\right)^{1/2}$$

$$= \left(\frac{1}{8\pi^2}\right)^{1/2}\times10^{-5}$$

$$= 0.11\times10^{-5}$$

$$= 1.1\times10^{-6}\ \text{m}$$

$$\delta = 1.1\ \mu\text{m}$$

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Theory Contact Continues :-

For bad conductor or poor conductor $\sigma << \epsilon\omega$ even ~~tol~~ at low frequency and $\alpha \simeq \omega\sqrt{\epsilon\mu}$ and $\beta = \frac{\sigma}{2}\sqrt{\mu/\epsilon}$.

and skin depth δ ~~is~~

$$\delta = \frac{1}{\beta} = \frac{2}{\sigma}\sqrt{\epsilon/\mu}$$

$$\alpha = k_0\left[\frac{+1 + \sqrt{1+(\sigma/\epsilon\omega)^2}}{2}\right]^{1/2}$$

$$= k_0\left[\frac{1 + 1 + \frac{1}{2}(\sigma/\epsilon\omega)^2}{2}\right]^{1/2}$$

$$= k_0\left[\frac{2 + \frac{1}{2}\left(\frac{\sigma}{\epsilon\omega}\right)^2}{2}\right]^{1/2}$$

for bad conductor

as $\sigma/\epsilon\omega^2 << 1$,

$$\alpha \simeq k_0 \cdot 1$$

$$= \sqrt{\mu\epsilon\omega^2} \cdot 1$$

$$\alpha = \omega\sqrt{\mu\epsilon}.$$

So in case of bad conductor the skin depth is independent of freq.

So in bad conductor the wave propagates without much loss.

In conductor the Maxwell equations are

i) $\vec{\nabla}\cdot\vec{E} = 0$

ii) $\vec{\nabla}\cdot\vec{B} = 0$

iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

& iv) $\vec{\nabla}\times\vec{B} = \mu\vec{J}_f + \mu\epsilon\frac{\partial\vec{E}}{\partial t}$

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In Conductor the em wave is damped as $\vec{k}$ is complex quantity and electric wave is represented by

$$\vec{E}(\vec{r},t) = \hat{\epsilon}_1 E_0 e^{j(\vec{k}\cdot\vec{r}-\omega t)}$$

$$= \hat{\epsilon}_1 E_0 e^{-\beta(\hat{\epsilon}_k\cdot\vec{r})} e^{+j\alpha(\hat{\epsilon}_k\cdot\vec{r}-\omega t)}$$

Where, $\vec{k} = \alpha + j\beta$.

we also find that here $\hat{\epsilon}_k\cdot\vec{E} = 0 = \hat{\epsilon}_k\cdot\vec{B}$ as inside the Conductor $\nabla\cdot\vec{E} = 0 = \nabla\cdot\vec{B}$ & $\nabla \equiv j\vec{k}$,

So the electric and magnetic field vectors are also perpendicular to propagation vector i.e. transverse inside the Conductor.

From 3rd eq^n (Maxwell)

$$\nabla\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

or $j\vec{k}\times\vec{E} = j\omega\vec{B}$ $\left[\begin{array}{l} \text{as } \nabla \equiv j\vec{k} \\ \frac{\partial}{\partial t} = -j\omega \end{array}\right]$

or $\vec{B} = \dfrac{\vec{k}\times\vec{E}}{\omega}$

$= \dfrac{\hat{\epsilon}_k\times\vec{E}}{\omega}(\alpha+j\beta)$. $\left[\because \vec{k} = (\alpha+j\beta)\hat{\epsilon}_k\right.$

$$\vec{B} = \frac{\hat{\epsilon}_k\times\vec{E}}{\omega}\sqrt{\alpha^2+\beta^2}\cdot e^{j\phi}$$

where $\phi = \tan^{-1}\beta/\alpha$.

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This shows that there exist a phase difference $\phi = \tan^{-1} \beta/\alpha$ between the $\vec{E}$ and $\vec{B}$ field. In fact the magnetic field lags behind $\vec{E}$. For a good conductor $\alpha \simeq \beta$. and phase lag is 45°.

The ratio

$$\left|\frac{\vec{B}}{\vec{E}}\right| = \frac{\sqrt{\alpha^2+\beta^2}}{\omega}$$

$$= \frac{1}{\omega} k_0 \left\{ 1 + \sqrt{1+(\sigma/\epsilon\omega)^2} \right\}^{1/2}$$

For a good conductor $\frac{\sigma}{\epsilon\omega} \gg 1$.

$$\therefore \left|\frac{\vec{B}}{\vec{E}}\right| = \frac{1}{\omega} \cdot \sqrt{\mu\epsilon\omega^2} \sqrt{\frac{\sigma}{\epsilon\omega}}$$

$$= \sqrt{\frac{\mu\sigma}{\omega}}$$

$$\therefore \left|\frac{\vec{B}}{\vec{E}}\right|^2 = \frac{\mu\sigma}{\omega}.$$

The ratio of average magnetic energy density and average electric energy density

$$\frac{\overline{U_m}}{\overline{U_e}} = \frac{\overline{B}^2/2\mu}{\frac{1}{2}\epsilon \overline{E}^2}$$

$$= \frac{1}{\mu\epsilon} \left|\frac{B}{E}\right|^2$$

$$= \frac{1}{\mu\epsilon} \cdot \frac{\mu\sigma}{\omega} = \frac{\sigma}{\epsilon\omega} \gg 1.$$

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So for good conductor the magnetic energy density $\overline{u}_m$ is very greater than electric energy density i.e

$$\overline{u}_m \gg \overline{u}_e .$$

So when an em wave travel through a conductor (good) the magnetic energy density $\overline{u}_m$ dominates over electric energy density.

Avarage Pointing vector in a Conductor

The avarage pointing vector of em-field in conductor

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu}$$

Again $$\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}.$$

$$= \frac{\hat{\varepsilon}_K\times\vec{E}}{\omega}(\alpha+j\beta).$$

So the avarage pointing vector

$$\langle\vec{S}\rangle = \frac{1}{2\mu}\,Re\; E\times\vec{B}^*$$

$$= \frac{1}{2\mu\omega}\,Re\;(E\times(\hat{\varepsilon}_K\times\vec{E}^*))(\alpha+j\beta).$$

$$= \frac{1}{2\mu\omega}\,Re\left\{\hat{\varepsilon}_K(\vec{E}\cdot\vec{E}^*) - \vec{E}^*\cancel{(\vec{E}\cdot\hat{\varepsilon}_K)}\right\}(\alpha+j\beta)$$

$$= \frac{1}{2\mu\omega}\,Re\left\{\hat{\varepsilon}_K\;\vec{E}\cdot\vec{E}^*\right\}(\alpha+j\beta)$$

$$= \hat{\varepsilon}_K\cdot\frac{1}{2\mu\omega}\,Re\;|E_0|^2(\alpha+j\beta)\,e^{-2\beta(\hat{\varepsilon}_K\cdot r)}.$$

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$$\langle \vec{S} \rangle = \hat{\varepsilon}_k \frac{1}{2\mu\omega} \sqrt{\alpha^2+\beta^2}\, Re\, |E_0|^2 e^{j\phi} \times e^{-2\beta(\hat{\varepsilon}_k\cdot\vec{r})}$$

$$= \hat{\varepsilon}_k \frac{1}{2\mu\omega} \sqrt{\alpha^2+\beta^2} \cos\phi\, |E_0|^2 e^{-2\beta(\hat{\varepsilon}_k\cdot\vec{r}-\cancel{\omega t})}$$

Now for good conductor

$$\alpha = \sqrt{\frac{\mu\omega\sigma}{2}}$$

$$\cos\phi = \frac{\alpha}{\sqrt{\alpha^2+\beta^2}} \left[\because \tan\phi = \beta/\alpha\right]$$

$$\langle \vec{S} \rangle = \hat{\varepsilon}_k \frac{\alpha}{2\mu\omega} |E_0|^2 e^{-2\beta(\hat{\varepsilon}_k\cdot\vec{r}-\cancel{\omega t})}$$

$$= \hat{\varepsilon}_k \frac{1}{2\mu\omega} \sqrt{\frac{\mu\omega\sigma}{2}} \cdot |E_0|^2 e^{-2\beta(\hat{\varepsilon}_k\cdot\vec{r}-\cancel{\omega t})}$$

$$= \hat{\varepsilon}_k \frac{1}{2} \sqrt{\frac{\sigma}{2\mu\omega}} |E_0|^2 e^{-2\beta(\hat{\varepsilon}_k\cdot\vec{r}-\cancel{\omega t})}$$

$$\langle \vec{S} \rangle = \hat{\varepsilon}_k \sqrt{\frac{\sigma}{2\mu\omega}} |E_{rms}|^2 e^{-2\beta(\hat{\varepsilon}_k\cdot\vec{r}-\cancel{\omega t})}$$

$$\left[\because E_{rms} = \frac{E_0}{\sqrt{2}}\right]$$

This equation shows that the energy flowing per sec per unit area normally decreases rapidly with the penetrating distance $(\hat{\varepsilon}_k\cdot\vec{r})$ due to the term $e^{-2\beta(\hat{\varepsilon}_k\cdot\vec{r}-\cancel{\omega t})}$

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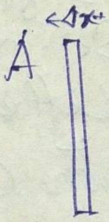
The power delivered to a conductor of volume V in the form of heat

$$P = \int_V (\vec{E}\cdot\vec{J})\, dv$$

Now for a conductor of area A and thickness Δx.



$$P = (\vec{E}\cdot\vec{J}) A\, \Delta x$$

$$= \sigma E^2 A\, \Delta x$$

The average power or average heat developed is

$$\langle P\rangle = \sigma A \Delta x\, \overline{E^2}$$

$$= \sigma A \Delta x \left(\frac{\vec{E}\cdot\vec{E}^*}{2}\right)$$

[by complex method of averaging]

$$\langle P\rangle = \frac{\sigma A\, \Delta x}{2} |E_0|^2 e^{-2\beta(\hat{\xi}_k\cdot\vec{r} - \cancel{\omega t})}.$$

So the average power for unit volume is

$$= \frac{\sigma}{2} |E_0|^2 e^{-2\beta(\hat{\xi}_k\cdot\vec{r} - \cancel{\omega t})}.$$

So the ~~avrg~~ average power is decreases with the penetrating distance, inside the conductor..

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Metalic Reflection :-

When a plane em wave from a dielectric medium is incident ~~on~~ on a ~~surface~~ surface on a conductor, reflection & refraction occurs quiet different form that by a non-conductor - non conductor boundary. In a conductor the constant ϵ, $\vec{k}$ and n are all replaced by corresponding complex form. The metals are good conductor of electricity and they are also good reflector of em wave.

The Snell's law of refraction by conducting medium is

$$\frac{\sin\theta_1}{\sin\theta_2} = \hat{n} \text{ (complex)}$$

$$\text{or } \sin\theta_2 = \frac{\sin\theta_1}{\hat{n}}$$

$$= \frac{\sin\theta_1}{n(1+j\chi)}.$$

where $\hat{n} = n(1+j\chi)$ and $\chi = \frac{\beta}{\alpha}$ is attenuation index.

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This means the angle of refraction in a conductor is complex number and it does not have a simple significance.

The Fresnel's formulae for conductor are still applicable

$$E_{10}' = -\left(\frac{\alpha-\beta}{\alpha+\beta}\right)E_{10}$$

where E_{10}' and E_{10} is the amplitude of reflected and incident light em wave. and $\alpha = \frac{\cos\theta_2}{\cos\theta_1}$ and $\beta = n_2/n_1 = n$, relative refractive index of 2 with r. to 1.

~~For normal incidence~~

and $$E_{20} = \left(\frac{2}{\alpha+\beta}\right)E_{10}$$

where E_{20} is the amplitude of the transmitted em wave.

For normal incidence $\alpha \simeq 1$. and the parameter $\beta = \tilde{n}$ The refractive index of the conductor relative to outside,

Therefore $$E_{10}' = -\left(\frac{1-\tilde{n}}{1+\tilde{n}}\right)E_{10}$$

For good conductor $\frac{\sigma}{\epsilon\omega} > 1$.

For a perfect conductor $\sigma \to \infty$

and $\tilde{n} \to \infty$

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So, $E_{10}' = E_{10}$

& $E_{20} = 0$

This shows the incidence electromagnetic wave is completely reflected and is out of phase.

For example, Ag, Au are good conducting reflector of 94–95% of incident light wave.

For a good conductor the reflection coefficient is (part of incident light wave reflected)

$$R = \left|\frac{E_{10}'}{E_{10}}\right|^2$$

$$= \left|\frac{1-\beta}{1+\beta}\right|^2$$

$$= \left|\frac{1-1/\beta}{1+1/\beta}\right|^2$$

$$= \left|(1-1/\beta)(1+1/\beta)^{-1}\right|^2$$

$$= \left|(1-1/\beta)^2\right|^2 \quad [\text{as } \beta \text{ is large}]$$

$$= \left|1-2/\beta+1/\beta^2\right|^2$$

$$= \left|1-2/\beta\right|^2 \quad \left[\frac{1}{\beta^2} \ll 1\right]$$

$$= (1-2/\beta)(1-2/\beta^*)$$

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$$= 1 - 2\left(\frac{1}{\beta} + \frac{1}{\beta^*}\right)$$

$$= 1 - 2\left(\frac{\beta + \beta^*}{\beta\beta^*}\right)$$

$$= 1 - 2 \cdot \left(\frac{\mu_2 \omega}{\mu_1 v_1}\right)\left(\frac{k_2 + k_2^*}{|k_2|^2}\right)$$

as $\left[\because \beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 v_1 \tilde{k}_2}{\mu_2 \omega}\right]$

$$= \cancel{1 - 2\,\frac{\mu_2 \omega}{\mu_1 v_1}\sqrt{\frac{2}{\mu_2 \sigma \omega}}}$$

$\left[\text{as } k_2 + \tilde{k}_2^* = 2\,\mathrm{Re}\,k_2 = 2\sqrt{\frac{\mu_2 \sigma \omega}{2}} = \sqrt{2\mu_2 \sigma \omega}\right.$

$\left.\text{and } k_2 \tilde{k}_2^* = |\tilde{k}_2|^2 = \mu_2 \sigma \omega.\right]$

$$R = 1 - 2\left(\frac{\mu_2 \omega}{\mu_1 v_1}\right) \cdot \frac{\sqrt{2\mu_2 \sigma \omega}}{\mu_2 \sigma \omega}$$

$$R = 1 - 2\left(\frac{\mu_2 \omega}{\mu_1 v_1}\right)\sqrt{\frac{2}{\mu_2 \sigma \omega}}$$

$$= 1 - \sqrt{\frac{8 \mu_2 \omega \epsilon_1}{\mu_1 \sigma}}.$$

$\left[v_1 = \frac{1}{\sqrt{\mu_1 \epsilon_1}}\right]$

As $\mu_1 \simeq \mu_2 \simeq \mu_0$ (non-ferromagnetic substance).

$$\therefore R = 1 - 2\sqrt{\frac{2\omega \epsilon_1}{\sigma}}$$

As σ is very high for good conductor $\simeq 10^7$. So $R \simeq 1$.

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So the reflectivity / reflection coefficient is nearly equals to 1 for good conductor.

The copper, and gold appears yellow with reflected light because the reflecting power for this colour is high. But when thin film of Cu and Au are seen by transmitted light, it will appear green.

* P.T at normal incidence the reflection coefficient is same for parallel & perpendicular to the plane of incidence polarized light wave.

We know ~~at normal incid~~ that for parallel polarised incident wave the expression of reflection coefficient

$$R = \left|\frac{\alpha - \beta}{\alpha + \beta}\right|^2$$

and for perpendicular incident wave

$$R' = \left|\frac{1 - \alpha\beta}{1 + \alpha\beta}\right|^2$$

So at normal incidence

$\alpha \simeq 1, \quad \beta = n = n_2/n_1$

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$$\therefore \quad R = \left|\frac{1 - n_2/n_1}{1 + n_2/n_1}\right|^2 = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2$$

$$R' = \left|\frac{1 - n_2/n_1}{1 + n_2/n_1}\right|^2 = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2$$

So $R = R'$ at normal incidence

(Proved)